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WAR DEPARTMENT

TECHNICAL MANUAL

MATHEMATICS FOR PILOT TRAINEES

April 22, 1942

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No. 1–900

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MATHEMATICS FOR PILOT TRAINEES

		Paragraphs
SECTION I.	General	1–2
II.	Fundamental operations	3–18
III.	Equations and formulas	19–22
IV.	Scales	23–26
V.	Graphs	27–29
VI.	Angular measurement	30–33
VII.	Vectors	34–37
		Page
APPENDIX.	Miscellaneous units and conversion factors	60
INDEX		62

SECTION I

GENERAL

	Paragraph
Purpose and scope	1
Materials	2

1. Purpose and scope.—*a.* The purpose of this manual is to provide, in convenient form, a review of some topics of arithmetic and related material which the pilot trainee must understand in order to practice simple air navigation and to cope with other problems of the practical airman.

b. Briefly, the scope includes the elementary operations of arithmetic, such as addition, subtraction, multiplication, and division, percent, ratio and proportion, angular measurements, scales, the use of graphs and formulas, and the graphical solution of the more common problems involving the triangle of velocity.

c. In mathematics, as in learning to fly, no amount of reading can replace actual practice. For this reason, many exercises have been included in each paragraph, and at the end of each section a collection of miscellaneous exercises has been added based on the material considered in that section. It is not contemplated that every student will do all of the exercises. However, an ample number of exercises has been inserted to provide an opportunity for those trainees who may feel the need for extra practice. The answers to the even numbered exercises are given to enable each student to check his own work

if he wishes. Illustrative examples are profuse and should help to clarify difficult points which may arise.

d. Undoubtedly, some of the topics will seem very simple to many of the trainees. It must be remembered, however, that the mathematical proficiency demanded of a pilot not only involves an understanding of the various operations, but also the ability to perform these operations *accurately* and *quickly*, and often under trying circumstances. Therefore the time spent in practicing such a simple operation as addition, for example, will not necessarily be so much time wasted—no matter how clearly the process is understood.

2. Materials.—In addition to pencil and paper, the student will need a ruler, a protractor, and a few sheets of graph paper.

Section II

FUNDAMENTAL OPERATIONS

	Paragraph
Purpose and scope	3
Addition	4
Subtraction	5
Multiplication	6
Division	7
Conversion of decimal fractions to common fractions	8
Conversion of common fractions to decimal fractions	9
Addition and subtraction of fractions	10
Multiplication of fractions	11
Division of fractions	12
Ratio and proportion	13
Positive and negative numbers	14
Addition of positive and negative numbers	15
Subtraction of positive and negative numbers	16
Multiplication and division of positive and negative numbers	17
Miscellaneous exercises	18

3. Purpose and scope.—The purpose of this section is to provide a review of the four fundamental operations of arithmetic: addition, subtraction, multiplication, and division. They are called the fundamental operations because all other mathematical operations are based on them. Every calculation that is made must use one or more of these operations.

4. Addition.—*a.* Addition is the operation of finding the sum of two or more numbers. To add several numbers, place the numbers in a vertical column so that the decimal points are all in a vertical line. (When no decimal point is indicated, it is assumed to be on the right.) Then add the figures in the right-hand column and place the sum under this column. If there is more than one figure in this sum,

write down only the right-hand figure and carry the others to the next column to the left.

(1) *Example:* Find the sum of 30.53, 6.475, 0.00035, and 3476.
Solution:

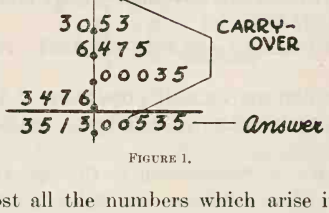


FIGURE 1.

b. Units.—Almost all the numbers which arise in practical arithmetic have to do with definite quantities such as 78 *feet*, 239 *miles*, 25 *degrees*, 160 *miles per hour*, 210 *pounds*, and so on. In these examples, the words in italics, which state what the quantities are in each case, are called the *units*. When adding several quantities together, it is clear that the units must all be the same. For example, "the sum of 78 *feet* and 160 *miles per hour*" is a completely meaningless statement.

(1) Units are so important and occur so often that standard abbreviations have been adopted for them. A list of the correct abbreviations and the relations which exist between some of the units are given in the appendix to this manual.

(2) *Example:* Find the sum of 78 feet and 3 miles.

Solution: In this case since 1 mile is the same as 5,280 feet (see appendix), then 3 miles is the same as 15,840 feet. Therefore 78 feet and 15,840 feet may be added together to give 15,918 feet. But the student is cautioned that unless there is a relation between the various units so that all the quantities may be expressed in terms of the *same* units, the addition *cannot* be performed.

c. Symbols.—In arithmetic and in other branches of mathematics, much space and effort is saved by using symbols. Thus, in order to write "find the sum of 70.765 and 301.4", the *plus sign* (+) is used and then this phrase can be written simply as "70.765+301.4=?." When more than two numbers are to be added, the plus sign is repeated: 70.765+301.4+765.84=1,138.005, for example.

d. Exercises.

(1) 30.53 in.+6.475 in.=?

(2) 648.03 cm+37.895 cm+219.921 cm+.08376 cm=
905.92976 cm. *Answer.*

(3) 100.001+9.098+5678.91=?
(4) 897.1+0.989+900.76+91901.359=93700.208 *Answer.*
(5) 9876 ft.+101.109 ft.+77.007 ft.+92.928 ft.+94.987 ft.+60.768 ft.=?
(6) 19.767+43.542+76.305+58.143+13.25=211.007 *Answer.*
(7) 11.1111 miles+66.667 miles+1.222 miles+125.125 miles+375.375 miles=?
(8) 78.908+202.202+62.501+0.003594+75=418.614594 *Answer.*
(9) 7.8098+20.202+6.2501+000.3594+7.5=?
(10) 78.808 yd.+98.15 yd.+760 yd.+88199.76 yd.=89136.718 yd. *Answer.*

5. Subtraction.—*a.* Subtraction is the operation of finding the difference between two numbers. In order to subtract one number from another, write the smaller number below the larger so that the decimal points are in a vertical column. Beginning with the right column, subtract the figures in the smaller number from the corresponding figures in the larger number above them.

(1) *Example:* Subtract 765.3 from 986.7.
Solution: 986.7
765.3
221.4 *Answer.*

b. If, however, the figure in the number being subtracted is larger than the figure directly above it, it is necessary to borrow one unit from the next figure to the left.

(1) *Example:* Subtract 765.3 from 843.1.
Solution:

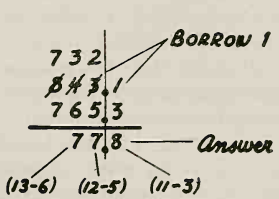


FIGURE 2.

(2) *Note.*—It is better to learn to do the "carrying over" mentally so that the preceding solution looks like this:

843. 1
765. 3
77. 8 *Answer.*

c. When a column has only one figure in it, zeros must be supplied in the blank spaces.

(1) *Example:* Subtract 765.328 from 843.1.
Solution:

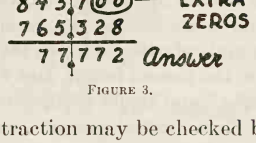


FIGURE 3.

d. A problem in subtraction may be checked by adding the answer to the number directly above it. The sum should *always* be the number in the top row.

(1) *Example:* Check the answer to the preceding example (5*c*(1)).
Solution:

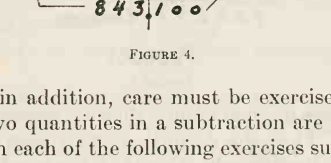


FIGURE 4.

(2) *Units.*—As in addition, care must be exercised to be sure that the units of the two quantities in a subtraction are the same.

e. Exercises.—In each of the following exercises subtract the smaller number from the larger. The symbol used to indicate subtraction is the *minus sign* (—). Therefore an expression such as "1,205.789—980.833=?" means "What is the remainder when 980.833 is subtracted *from* 1,205.789?". The number following the minus sign is always subtracted from the number preceding the sign.

(1) 1,205.789 in.—980.833 in.=?
(2) 19.52—.78=18.74 *Answer.*
(3) 760,591—674,892=?
(4) 73.44 cu. in.—8.7375 cu. in.=64.7025 cu. in. *Answer.*
(5) 89.73—10.0045=?
(6) 941.7—87.372=854.328 *Answer.*
(7) 1,004.78 miles—1,004.164 miles=?
(8) 100,433 sq. ft.—99,857 sq. ft.=576 sq. ft. *Answer.*
(9) 1,000,000.3—998,757.4=?
(10) 3,756.04—2,489.7=1,266.34 *Answer.*

6. Multiplication.—*a.* Multiplication is a short method of adding a number to itself a certain number of times. The numbers which are

multiplied together are called the *factors* and the result of the multiplication is called the *product*. To multiply two numbers together, or in other words, to find the product of two factors, first write the factors one below the other (see illustrative example). It is usually easier to operate with the smaller number of figures in the second row. Multiply the factor in the top row by the right-hand figure of the factor in the second row, and write this partial product directly under the second factor. Then multiply the factor in the top row by the second figure from the right in the second factor, and write this second partial product so that its right-hand figure is directly under the figure that was used to find it. These partial products are then added together to yield the required product.

(1) *Example:* Multiply 1,653 by 247.

Solution:

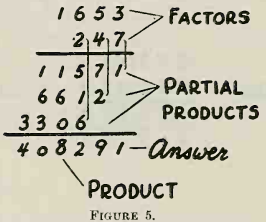


FIGURE 5.

b. When there are decimal points, they are ignored until the product has been found. Then the decimal point is inserted in the product according to the following rule: Count off the number of figures to the right of the decimal point in each factor. Then the number of figures to the right of the decimal point in the product is equal to the *sum* of the number of figures after the decimal point in each factor.

(1) *Example:* Multiply 16.53 by 24.7.

Solution:

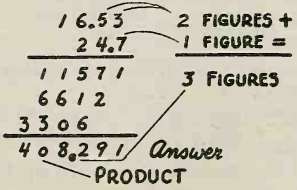


FIGURE 6.

c. When the second factor contains zeros, the partial products corresponding to these zeros need not all be written down. Only the right-hand zero is written down. However, care must be exercised to

have the right-hand figures of all the partial products directly below their corresponding figures in the second factor.

(1) *Example:* Multiply 16.53 by 24.07.

Solution:

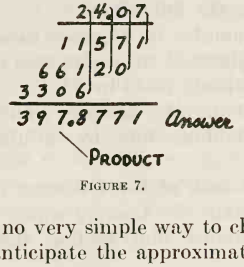


FIGURE 7.

d. Although there is no very simple way to check a multiplication, it is good practice to anticipate the approximate size of the product before beginning a long multiplication. This is done by "rounding-off" the factors to permit easy mental multiplication. Although by no means an accurate check, this will frequently catch mistakes in addition or in the location of the decimal point which would otherwise result in nonsensical answers.

(1) *Example:* What is the approximate product of 15.73 multiplied by 187.04?

Solution: Round-off 15.73 to 15, and 187.04 to 200. Then the product is roughly 15 by 200=3,000. It is clear then that the product of 15.73 and 187.04 cannot be 150.6 or 6,030.3745, for example.

e. Symbols and units.—The more common symbol for multiplication is $\times$. However, it is quite common simply to write the numbers in parentheses next to each other: $(3.04)(17.78)=3.04\times17.78=54.0512$, for example.

(1) When the same number is to be multiplied by itself, for example 3.04×3.04, this is usually indicated by a small 2 placed above and to the right of the number: $3.04\times3.04=3.04^2$. This is read as "3.04 *squared*," and 3.04^2 is the "*square* of 3.04." If the number is to be used as a factor 3 times, then a small 3 is used: $3.04\times3.04\times3.04=3.04^3$. This is read as "3.04 *cubed*," and 3.04^3 is the "*cube* of 3.04."

(2) Unlike addition and subtraction, different units may be multiplied together. The product is then expressed in a unit which is itself the product of the units of the factors.

(*a*) *Example:* Multiply 5 pounds by 7 feet.

Solution: (5 pounds)(7 feet)=35 pounds-feet=35 lb.-ft.=35 ft.-lb. *Answer.*

(*b*) *Example:* Multiply 9 feet by 17 feet.

Solution: (9 feet)(17 feet)=153 feet×feet=153 (feet)^2=153 ft.^{2} ("ft.^{2}" is read "square feet"). *Answer.*

f. In arithmetic, as in other operations, there are many tricks which often simplify the work. One such trick which is often useful and easy to remember is the following:

To multiply any number by 25, move over the decimal point in the given number two places to the right; then divide by 4.

(1) *Example:* Multiply 16.53 by 25.

Solution: Moving the decimal point over two places, 16.53 becomes 1653. Dividing this by 4: 1653/4=413.25. Therefore 16.53×25=413.25. *Answer.*

g. Exercises.—To each of the following exercises 3 answers have been given. Eliminate the answers which are obviously wrong by rounding-off the factors and finding the approximately correct answers mentally.

(1) 600.3×42.7= { 25632.81 / 1200.62 / 4273.21

(2) 180×76= { 30,740 / 13,680 / 25,580

(3) 12.45×18.3= { 400.785 / 60.785 / 227.835

(4) 88×3.2= { 472.6 / 281.6 / 31.7

(5) 1751.2×36.4= { 63,743.68 / 6,374.368 / 12,743.68

(6) 903×8.475= { 3652.925 / 7652.925 / 76.52925

h. Exercises.—Perform the indicated multiplications.

(1) .0734 in.×70. 34 in.=?

(2) 831.43×71.46=59,413.9878 *Answer.*

(3) 1.0073 in.×6.4 ft.=?

(4) 8.94×9.37=83.7678 *Answer.*

(5) 8,374.5×9,378.46=?

(6) 10,742 lb.×737.2 ft.=7,919,002.4 ft.-lb. *Answer.*

(7) 23.1×847.4=?

(8) 9,034×10.06=90,882.04 *Answer.*

(9) 8.037 ft.$\times$24.2 lb.=?

(10) .0074$\times$371.5=2.7491 *Answer.*

7. Division.—*a.* Division is the process of finding how many times one number is contained in another. The number to be divided is called the *dividend*, and the number by which it is divided is called the *divisor*. The result of the operation, or the answer, is called the *quotient*.

b. When the divisor contains but one figure, the method commonly used is known as *short* division. To perform short division, place the divisor (one figure) to the left of the dividend, separated by a vertical line (see illustrative example). Then place a horizontal line over the dividend. Divide the first or the first two figures of the dividend, as is necessary, by the divisor and place the quotient over the line. If the divisor does not go an even number of times, the remainder is prefixed to the next figure in the dividend and the process is repeated.

(1) *Example:* Divide 4,644 by 6.

Solution:

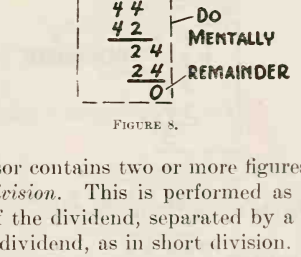


FIGURE 8.

c. When the divisor contains two or more figures, the method used is known as *long division*. This is performed as follows: Place the divisor to the left of the dividend, separated by a line, and place the quotient above the dividend, as in short division. Using the divisor, divide the first group of figures of the dividend which gives a number as large or larger than the divisor (see illustrative example). Place the first figure of the quotient above the dividend. Then multiply this figure by the divisor, and place the product below the figures of the dividend which were used for this division. Then subtract this product from the figures directly above it. The next figure in the original dividend is brought down to form a new dividend. This is repeated until all the figures of the original dividend have been used.

(1) *Example:* Divide 6,646,250 by 10,634.
Solution:

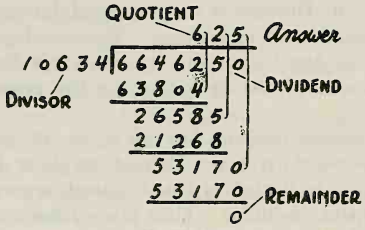


FIGURE 9.

d. It is not very common in both short and long division to have the divisor go into the last trial dividend a whole number of times. When the last trial remainder is not zero, it must be indicated in the answer.

(1) *Example:* Divide 4,647 by 6.
Solution:

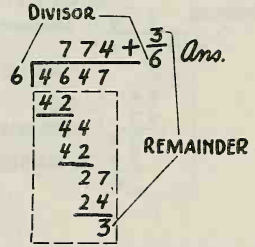


FIGURE 10.

(2) *Example:* Divide 6,646,257 by 10,634.
Solution:

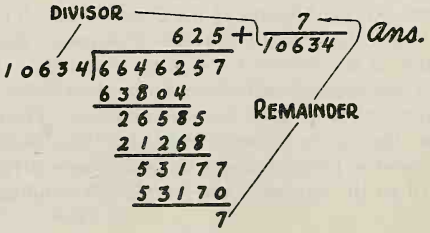


FIGURE 11.

e. Symbols.—"4,647 divided by 6" may be indicated in symbols in several ways. The division sign may be used: 4,647÷6. The "stroke" is more convenient to use on the typewriter; 4,647/6. Finally, the division may be indicated as a *fraction:* $\frac{4647}{6}$. The fact that 4,647 divided by 6 is 774 with a remainder of 3 may be written as "$4647 \div 6 = 774 + \frac{3}{6}$," or "$4{,}647/6 = 774 + \frac{3}{6}$," or "$\frac{4647}{6} = 774 + \frac{3}{6}$."

f. Decimal point.—To locate the decimal point in the quotient when decimal points are present in either the divisor or the dividend, do the following: Move the decimal point in the divisor to the right of the right-hand figure. Then move the decimal point in the dividend to the right the same number of places that the point was moved in the divisor. When dividing be careful to place the quotient so that each figure of the quotient is directly above the right-hand figure of the group of figures which were used in the dividend. Then the decimal point in the quotient will be directly above the new position of the decimal point in the dividend.

(1) *Example:* Divide 4.644 by .06.
Solution:

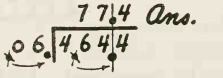


FIGURE 12.

(2) *Example:* Divide 6.646250 by 10.634.
Solution:

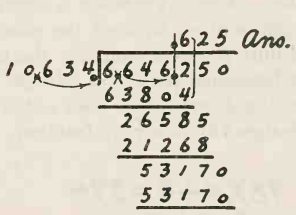


FIGURE 13.

g. If decimal points are involved in a division, it is not customary to indicate the remainder when the division does not come out "even". In lieu thereof extra zeros are added to the dividend, and the division is continued until the quotient has as many figures as desired. The number of figures to obtain in the quotient is largely a matter of common sense.

(1) *Example:* Find 46.47/0.6.
Solution:

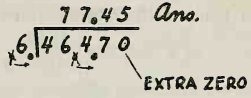


FIGURE 14.

(2) *Example:* Find 6.646250÷10.637.
Solution:

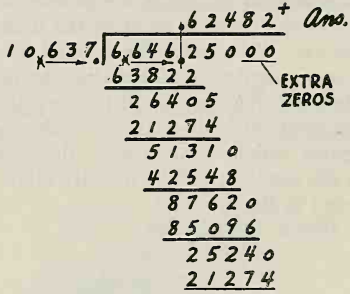


FIGURE 15.

h. Mixed numbers.—In paragraph 7*e* it was mentioned that "$\frac{4647}{6}=774+\frac{3}{6}$." When the plus (+) sign is omitted, then $774\frac{3}{6}$ is called a *mixed number.* A mixed number is simply the sum of a whole number and a fraction written without the plus sign. To convert a mixed number to a pure fraction, multiply the whole number by the denominator of the fractional part, and add the numerator. This is the new numerator; the denominator does not change.

(1) *Example:* Change $78\frac{4}{5}$ to a pure fraction.
Solution:

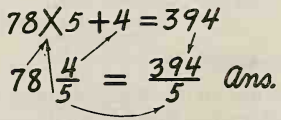


FIGURE 16.

i. Checks.—Any division may be checked by multiplying the divisor by the quotient and adding the remainder. The result is *always* the dividend.

(1) *Example:* Check the answer to example 7*g* (2).
Solution: $.6248 \times 10.637 = 6.6459976$.

$6.6459976 + \text{remainder} = 6.6459976$
$+.0002524$
6.6462500 *Check*

j. Units.—As in multiplication, any two different quantities may be divided, even though the units are not the same. The quotient is expressed in a unit which is itself the quotient of the units of the dividend and the divisor.

(1) *Example:* Divide 175 miles by 10 hours.

Solution: $\frac{175 \text{ miles}}{10 \text{ hours}} = 17.5 \frac{\text{miles}}{\text{hours}}$ *Answer.*

(In units of this sort it is customary to write the denominator in the singular, and use the stroke (/) to separate the numerator from the denominator: 17.5 miles/hour, or 17.5 mi./hr. Since "miles/hour" is read as "miles *per* hour," this phrase is usually abbreviated as "mph".)

(2) *Example:* Divide 500 pounds by 50 square inches.

Solution: $\frac{500 \text{ pounds}}{50 \text{ square inch}} = 10$ pounds/square inch $= 10$ lb./sq. in. *Answer.*

("10 pounds per square inch")

k. Exercises.—Perform the indicated divisions, and express the quotient as a mixed number.

(1) 894/16

(2) 755/24

Solution:
$$24\overline{)755}\quad 31$$
$$\underline{72}$$
$$35$$
$$\underline{24}$$
$$11$$

$31\frac{11}{24}$ *Answer.*

(3) 1,025/314

(4) 215/72 $2\frac{69}{72}$ *Answer.*

(5) 4,723/353

(6) 6,754,000/11,411 $591\frac{10099}{11411}$ *Answer.*

(7) 9,001/30

(8) 11,415/45 $253\frac{2}{3}$ *Answer.*

(9) 673/37

(10) 1,371/38 $36\frac{3}{38}$ *Answer.*

l. Exercises.—In the following exercises, express the quotient as a decimal. Do not obtain more figures after the decimal point than the number of figures after the decimal point in either the dividend or divisor.

(1) 73.01/3.4

(2) .345/.36

Solution:

$$\begin{array}{r} .958 \\ 36\overline{)34.500} \\ 32\,4 \\ \hline 2\,10 \\ 1\,80 \\ \hline 300 \\ 288 \end{array}$$

Since there are only 2 figures after the decimal point in .36, the quotient is "rounded-off" to .96. *Answer.*

If the figure to be thrown away is greater than or equal to 5, increase the figure on the left by one. If the figure is less than 5, do not change the preceding figure. Thus: $.953=.95$; $1.057=1.06$; $1.053=1.05$, etc.

(3) 13.37/.834

(4) 14.705/8.64 1.70 *Answer.*

(5) (157 miles)/(17.3 hours)

(6) 1,942.4/.0035 554971.4 *Answer.*

(7) 9.63/145.4

(8) (198 miles)/(59 minutes) 3.+miles/min. *Answer.*

(9) (5,280 feet)/(60 seconds)

(10) 19.437/38.6 .5 *Answer.*

8. Conversion of decimal fractions to common fractions.—*a.* A number which consists of a decimal point followed by a sequence of figures is called a *decimal fraction.* Thus .33, .9899, .00467, and .00335 are all decimal fractions. Since 33 divided by 100 is .33, then $.33=\frac{33}{100}$. Similarly, $.9899=\frac{9899}{10,000}$, and $.00467=\frac{467}{100,000}$. Therefore, to express any decimal fraction in fractional form, (1) write the number without the decimal point and (2) divide it by 1 followed by as many zeros as there are figures after the decimal point in the given number.

(1) *Example:* Express .023678 in fractional form.

Solution:

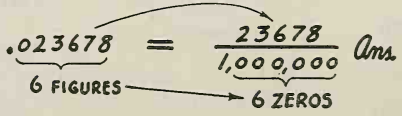


FIGURE 17.

(2) *Example:* Express 4.0785 in fractional form.
Solution: First express .0785 as a fraction.

$$.0785=\frac{785}{10{,}000}$$

Then $4.0785=4+\frac{785}{10{,}000}=4\frac{785}{10{,}000}$ *Answer.*

b. As already mentioned in paragraph 7*e*, a fraction such as 54/16 or $\frac{54}{16}$ is really just an indicated division. Very frequently in calculations it is much easier to carry a fraction along as a fraction than it is to "divide it out." Later on, this problem will be considered in detail. At present, however, there is one very important rule of operation on fractions which should be mastered.

(1) This rule is that both the dividend (numerator) and divisor (denominator) of any fraction may be divided or multiplied by any number (except zero), without changing the value of the fraction. For example, if the numerator (54) and the denominator (16) of the fraction $\frac{54}{16}$ are both divided by 2, then according to this rule $\frac{54}{16}=\frac{27}{8}$.

c. A fraction is said to be in its lowest terms or simplest form if there is no number which will divide both the numerator and denominator evenly. The operation of finding the simplest form of a fraction is called *reduction* to lowest terms or *simplification.*

(1) *Example:* Simplify $\frac{632}{32}$.

Solution: Divide numerator and denominator by 4:

$$\frac{632}{32}=\frac{158}{8}.$$

This can be simplified still more by dividing by 2:

$$\frac{158}{8}=\frac{79}{4}$$ *Answer.*

d. Exercises.—In the following exercises express the given decimal fractions in fractional form and then simplify. When possible, express the simplified fraction as a mixed number.

(1) 1.875=?

(2) .9375=? $\frac{15}{16}$ *Answer.*

(3) 2.109375=?

(4) .125=? $\frac{1}{8}$ *Answer.*

(5) .890625=?

(6) 3.828125=? $3\frac{53}{64}$ *Answer.*

(7) .625=?
(8) 4.375=? $4\frac{3}{8}$ *Answer.*
(9) 1.6875=?
(10) 2.3125=? $2\frac{5}{16}$ *Answer.*

e. Percent.—Percent means per hundred. Thus, for example, 6 percent interest means that the interest is 6 dollars per 100 dollars, or for each 100 dollars. In many applications it is often necessary to express a percentage as a decimal and conversely.

(1) To change from percent to a decimal, divide the number of percent by 100, which is equivalent to moving the decimal point two places to the left, and omit "percent."

(*a*) *Example:* Change 42 percent to a decimal.
Solution: 42 percent=42/100=.42 *Answer.*

(*b*) *Example:* Change .9 percent to a decimal.
Solution: .9 percent=.9/100=.009 *Answer.*

(2) To change from a decimal to a percent, multiply the decimal fraction by 100, which is equivalent to moving the decimal point two places to the right, and add "percent" (or "%").

(*a*) *Example:* Express .45 as a percent.
Solution: .45 × 100=45 percent *Answer.*

(*b*) *Example:* Express 6.47 as a percent.
Solution: 6.47 × 100=647 percent *Answer.*

(3) To find a certain percent of a given number, express the percent in decimal form and then multiply.

(*a*) *Example:* What is 6.2 percent of 115?
Solution: 6.2 percent=.062.
6.2 percent of 115=.062 × 115=7.130 *Answer.*

(*b*) *Example:* 69 percent of the airplanes out of a squadron of 13 airplanes are available for combat. What number of airplanes of the squadron are available for combat?
Solution: 69 percent=.69
.69 × 13 airplanes=9 airplanes *Answer.*

(4) It is common to specify changes in troop strength, population, enlistments, and even changes in the physical dimensions of pistons, for example, in terms of percent. The following examples illustrate the methods of solving this kind of problem.

(*a*) *Example:* If the population of the United States was 120,000,000 in 1930 and increased 7 percent during that year, what was it in 1931?
Solution: 7 percent=.07
.07 × 120,000,000=8,400,000.
Therefore the population in 1931 was 120,000,000+8,400,000=128,400,000 *Answer.*

(*b*) *Example:* At a certain altitude and temperature, the true air speed is 14 percent greater than the calibrated airspeed. If the calibrated air speed is 172 mph, what is the true air speed?

1st solution: 14 percent=.14
.14 × 172 mph=24 mph
Therefore 172 mph+24 mph=196 mph *Answer.*
2d solution 114 percent=1.14
1.14 × 172 mph=196 mph *Answer.*

(*c*) *Example:* At a certain altitude and temperature, the true air speed is 14 percent greater than the calibrated air speed. If the true air speed is 210 mph, what is the calibrated air speed?

Solution: Caution must be used in this type of problem. Percent increases as usually understood mean so many percent of the *smaller* number. Consequently, since 14 percent=.14,

calibrated air speed+(.14)×(calibrated air speed)=210 mph
Therefore (1.14) (calibrated air speed)=210 mph
or calibrated air speed=(210 mph)/1.14
=184 mph *Answer.*

This kind of problem can be checked very easily. 14 percent of calibrated air speed=.14×184=26 mph, so that 14 percent of calibrated air speed+calibrated air speed=26 mph+184 mph=210 mph. *Check.*

f. Exercises.

(1) 23 percent of a class of 1,900 cadets attend two-engine training schools. Find the number of cadets in this class who attended these schools.

(2) 36 percent of a class of 1,800 cadets take their advanced training in Alabama. How many cadets does this represent? 648 *Answer.*

(3) A lieutenant calls a detail of 24 men. This represents 6 percent of the men in his squadron. How many men are there in the squadron?

(4) At 1,000 rpm a propeller uses 80 percent of the horsepower developed. If the engine develops 1,500 horsepower at 1,000 rpm, how many horsepower are used by the propeller? 1,200 hp *Answer.*

(5) The top speed of an aircraft at 8,000 ft. is 314 mph. At 12,000 ft. the top speed has increased 8 percent. What is the top speed of the aircraft at 12,000 ft.?

(6) For a certain air density, the true altitude is 16 percent greater than the calibrated altitude. If the true altitude is 17,400 ft., find the calibrated altitude. 15,000 ft. *Answer.*

(7) At 15,000 ft. altitude and at −10° C., the calibrated air speed is 240 mph. The true air speed is 15 percent more than the calibrated air speed. Find the true air speed.

(8) A sample of nickel steel contained 25.61 percent of nickel and 0.17 percent of carbon. How much nickel and how much carbon would be found in a ton of this nickel steel?

512.2 lb. nickel
3.4 lb. carbon *Answer.*

(9) A machine shop employing 225 men is forced to employ 36 percent more men. What is the increase in the number of employees?

(10) The indicated horsepower of an engine is 1,500. The actual effective horsepower is 16 percent less than the indicated horsepower. Find the actual effective horsepower.

1,260 hp *Answer.*

9. Conversion of common fractions to decimal fractions.—*a.* To convert any fraction to a decimal form, simply add a decimal point on the right of the numerator and perform the indicated division as in paragraph 7*g*.

(1) *Example:* Express $\frac{5}{8}$ as a decimal fraction.

Solution: $8/\overline{5.0000}$ $\quad .6250$. Therefore $\frac{5}{8}=.625$ *Answer.*

(2) *Example:* Express $\frac{19}{4}$ as a decimal.

Solution: $4/\overline{19.000000}$ $\quad 4.7500$. Therefore $\frac{19}{4}=4.75$ *Answer.*

b. To express any fraction as a percent, first change the fraction to a decimal fraction as in the preceding paragraph, then change the decimal fraction to a percent as in paragraph 8.

(1) *Example:* Express $\frac{7}{12}$ as a percent.

Solution: $12/\overline{7.000000}$ $\quad .583333...$ Therefore $\frac{7}{12}=.583333...$

(The series of dots indicates that the decimal fraction may be continued indefinitely by adding 3's.)

Therefore $\frac{7}{12}=.583333...=58.3$ percent *Answer.*

c. Percent problems.—The following examples illustrate the converse problems to those given in paragraph 8.

(1) *Example:* If 18 airplanes out of a squadron of 27 airplanes are available for combat, what percent of the squadron aircraft are available for combat? What percent are not available for combat?

Solution: $\frac{18}{27}=.6666...=67$ percent available for combat *Answer.*

$100-67$ percent $=33$ percent not available for combat,

or $\frac{9}{27}=.333...=33$ percent *Answer.*

(2) *Example:* At a pressure altitude of 20,000 ft. with the air temperature at 10° below zero, the calibrated air speed is 200 knots, and the true air speed is 282 knots. What is the percent increase in the air speeds? Compare the true air speed with the calibrated air speed.

Solution: 282 knots—200 knots=82 knots.

82 knots/200 knots=.41=41 percent increase *Answer.*

282 knots/200 knots=1.41=141 percent. Therefore the true air speed is 141 percent of the calibrated air speed. *Answer.*

d. Exercises.

(1) The top speed of an aircraft at 8,000 ft. is 304 mph. At 12,000 ft. the top speed is 352 mph. What is the percent of increase in speed?

(2) The indicated horsepower of an engine is 1,500, while the actual effective horsepower is 1,275. What percent of the indicated horsepower is the actual horsepower? 85 percent *Answer.*

Express each of the following as a decimal fraction:

(3) $\frac{7}{8}$

(4) $\frac{5}{12}$ .4166... *Answer.*

(5) $\frac{9}{16}$

(6) $\frac{3}{32}$ .09375 *Answer.*

Express each of the following as a percent.

(7) $\frac{1}{4}$

(8) $\frac{3}{8}$ 37.5 percent *Answer.*

(9) $\frac{2}{5}$

(10) $\frac{7}{16}$ 43.75 percent *Answer.*

10. Addition and subtraction of fractions.—*a.* When several fractions are to be added which all have the same denominator, the addition is performed by simply adding the numerators.

(1) *Example:* $\frac{5}{7}+\frac{8}{7}+\frac{3}{7}+\frac{10}{7}=?$

Solution: $\frac{5}{7}+\frac{8}{7}+\frac{3}{7}+\frac{10}{7}=\frac{5+8+3+10}{7}=\frac{26}{7}$ *Answer.*

b. When one fraction is to be subtracted from another, and both have the same denominator, the subtraction is performed by simply subtracting the numerators.

(1) *Example:* $\frac{9}{11}-\frac{6}{11}=?$

Solution: $\frac{9}{11}-\frac{6}{11}=\frac{9-6}{11}=\frac{3}{11}$ *Answer.*

c. When fractions to be added or subtracted do not all have the same denominator, the fractions must first be reduced to the same common denominator. What the common denominator is does not matter very much. The reduction to the common denominator is performed by applying the rule given in paragraph 8*b*.

(1) *Example:* $\frac{9}{11}+\frac{7}{10}+\frac{4}{5}=?$

Solution: By inspection it is found that all the fractions can be reduced to a denominator=110. The first is multiplied by 10 (both numerator and denominator), the second by 11, and the third by 22.

Then $\frac{9\times10}{11\times10}=\frac{90}{110}$; $\frac{7\times11}{10\times11}=\frac{77}{110}$; and $\frac{4\times22}{5\times22}=\frac{88}{110}$

therefore $\frac{9}{11}+\frac{7}{10}+\frac{4}{5}=\frac{90}{110}+\frac{77}{110}+\frac{88}{110}=\frac{90+77+88}{110}=\frac{255}{110}$

but $\frac{255}{110}=\frac{255/5}{110/5}=\frac{51}{22}$ *Answer.*

(2) The easiest way to find a common denominator is to multiply all the different denominators together. This will frequently give a common denominator which is larger than necessary, but if the answer is then reduced to lowest terms this method will always work.

(3) *Example:* $\frac{19}{25}-\frac{16}{33}=?$

Solution: $25\times33=825$

Then $\frac{19}{25}-\frac{16}{33}=\frac{627}{825}-\frac{400}{825}=\frac{627-400}{825}=\frac{227}{825}$ *Answer.*

d. Exercises:

(1) $\frac{2}{3}-\frac{1}{3}=?$

(2) $\frac{7}{8}-\frac{3}{4}=\frac{1}{8}$ *Answer.*

(3) $\frac{11}{16}-\frac{1}{2}=?$

(4) $\frac{2}{3}+\frac{3}{4}+\frac{5}{6}+\frac{7}{8}+\frac{11}{12}+\frac{13}{12}=5\frac{1}{8}$ *Answer.*

(5) $9\frac{1}{6}+8\frac{5}{3}+7=?$

Hint: Add $\frac{1}{6}$ and $\frac{5}{3}$ first.

(6) $7\frac{2}{3}+9\frac{3}{4}+11\frac{1}{2}=?$ $28\frac{11}{12}$ *Answer.*

(7) $14\frac{3}{4}+30\frac{1}{2}+4=?$

(8) A dealer had 16 gallons of oil to sell. He sold $1\frac{1}{2}$ gallons to one customer, $2\frac{3}{4}$ gallons to another, $7\frac{1}{4}$ gallons to another, and the remainder to a fourth customer. How much did he sell to the fourth customer?

$4\frac{1}{2}$ gallons *Answer.*

(9) A man can do a piece of work in $13\frac{7}{8}$ days. A boy can do the same piece of work in $19\frac{1}{2}$ days. How much longer does it take the boy to do the work?

(10) The distance from outside to outside between two holes in a steel plate is $6\frac{2}{3}$ inches. If one hole is $1\frac{1}{6}$ inches in diameter and the other is $2\frac{1}{24}$ inches in diameter, find the length of metal between the holes.

$3\frac{11}{24}$ inches *Answer.*

11. Multiplication of fractions.—*a.* Two or more fractions are multiplied by multiplying the numerators together, and multiplying the denominators together. The product is then a fraction whose numerator is the product of the several numerators, and whose denominator is the product of the denominators.

(1) *Example:* Multiply $\frac{8}{9}$ by $\frac{17}{15}$.

Solution:

$$\frac{8}{9}\times\frac{17}{15}=\frac{8\times17}{9\times15}=\frac{136}{135}=1\frac{1}{135}\ \text{Ans.}$$

FIGURE 18.

b. To multiply mixed numbers first change the mixed numbers to fractions, and then multiply as in the preceding paragraph.

(1) *Example:* Multiply $15\frac{2}{3}$ by $19\frac{2}{5}$.

Solution: $15\frac{2}{3}=\frac{47}{3}$ and $19\frac{2}{5}=\frac{97}{5}$.

$\frac{47}{3}\times\frac{97}{5}=\frac{47}{3}\times\frac{97}{5}=\frac{4559}{15}=303\frac{14}{15}$ *Answer.*

c. Exercises.

(1) $\frac{3}{4}\times 5=?$

(2) $\frac{6}{7}\times\frac{7}{6}=?$ 1 *Answer.*

(3) $12\times\frac{3}{4}=?$

(4) $63\times 2\frac{2}{9}=?$

Solution: $63\times 2\frac{2}{9}=63\times\frac{20}{9}=\frac{\overset{7}{\cancel{63}}}{1}\times\frac{20}{\cancel{9}}=140$ *Answer.*

(5) $12\frac{2}{3}\times 15\frac{2}{5}=?$

(6) A tank holds 300 gallons of gas. If a pipe empties $\frac{1}{4}$ of the gas in an hour, how many gallons will be left in the tank at the end of two hours?

Solution: $\frac{1}{4}\times 300=75$ gal./hr. In two hours, pipe will empty 150 gallons. Therefore 300—150 gallons will be left. 150 gallons. *Answer.*

(7) A tank is $\frac{5}{8}$ full of gas. If $\frac{1}{8}$ of this is drawn off, what part of the whole tank is drawn off? What part remains in the tank?

(8) The circumference of a circle is about $3\frac{1}{7}$ times the diameter. Find the circumference of a circle if the diameter is 14 feet; if 28 feet; if $\frac{1}{22}$ foot. 44 ft., 88 ft., $\frac{1}{7}$ ft. *Answers.*

(9) If a motor makes 2100 rpm, how many revolutions does it make in $\frac{3}{4}$ hour? In $3\frac{10}{21}$ days?

(10) An alloy, used for bearings in machinery, is $\frac{24}{29}$ copper, $\frac{4}{29}$ tin, and $\frac{1}{29}$ zinc. How many pounds of each in 346 pounds of the alloy?
286.34 lb. copper, 47.72 lb. tin, 11.93 lb. zinc *Answers.*

12. Division of fractions.—*a.* To divide one fraction by another first invert the divisor and then multiply as in paragraph 11.

(1) *Example:* Divide $\frac{8}{9}$ by $\frac{15}{17}$.

Solution:

FIGURE 19.

b. To divide mixed numbers, first change to a fractional form, then divide as in the preceding paragraph.

(1) *Example:* Divide $7\frac{2}{3}$ by $8\frac{3}{4}$.

Solution: $7\frac{2}{3} \div 8\frac{3}{4} = \frac{23}{3} \div \frac{35}{4} = \frac{23}{3} \times \frac{4}{35} = \frac{92}{105}$ *Answer.*

c. Exercises.

(1) $\frac{5}{7} \div 10 = ?$

(2) $\frac{1}{3} \div \frac{4}{3} = ?$ $\frac{1}{4}$ *Answer*

(3) $27\frac{3}{5} \div 9 = ?$

(4) If $\frac{1}{10}$ inch on a map represents 49 miles, how many miles are represented by 3 inches on the map?

Solution: $\frac{3 \text{ in.}}{\frac{1}{10} \text{ in.}} = 3 \times \frac{10}{1} = 30.$

Therefore 3 inches represents a distance 30 times as great as 49 miles= $30 \times 49 = 1{,}470$ miles *Answer.*

(5) In the blueprint of a house $\frac{1}{4}$ inch in the print represents 1 foot in the actual house. Find the dimensions of the rooms that measure as follows: $2\frac{1}{2}$ by $2\frac{1}{2}$ inches, $4\frac{1}{8}$ by $4\frac{5}{8}$ inches, $5\frac{3}{8}$ by 6 inches, $3\frac{1}{16}$ by $4\frac{1}{32}$ inches, respectively, on the blueprint.

(6) Two places A and B are 24 miles apart on a river that flows 3 miles an hour. A man can row 5 miles an hour in still water. He goes from A to B and back. Find the time for the journey.

Hint: Man's speed down river is 8 mph. 15 hr. *Answer.*

(7) A car is going 1.125 miles per hour. How long will it take this car to go $468\frac{3}{4}$ miles?

(8) A layer of No. 8 wire, which is 0.162 inch in diameter, is wound on a pipe $24\frac{3}{8}$ inches long. How many turns of wire are wound on the pipe? 150.46 turns *Answer.*

(9) A mechanic can assemble $\frac{1}{32}$ of a motor in one day. How many motors can he assemble in $3\frac{1}{2}$ days?

(10) If a pilot flies 347 miles in 3 hours, 15 minutes, how far will he travel at the same rate in 7 hours, 45 minutes?

827.46 mi. *Answer.*

13. Ratio and proportion.—*a.* Consider two bombs, one weighing 300 lb. and the other 100 lb. The first is 3 times as heavy as the second, or the second is $\frac{1}{3}$ as heavy as the first. This may be

expressed as "the *ratio* of the weight of the second bomb to the weight of the first bomb is $\frac{1}{3}$". In other words, *a ratio is the quotient of two like quantities.* In this example,

$$\text{ratio}=\frac{100 \text{ lb.}}{300 \text{ lb.}}=\frac{1}{3}$$

b. The statement that two ratios are equal is called a *proportion.* Thus, for example, if the explosive in the first bomb is 270 lb., and the explosive in the second bomb is 90 lb., then the ratios of the explosives are also $\frac{1}{3}$, and

$$\frac{100 \text{ lb.}}{300 \text{ lb.}}=\frac{90 \text{ lb.}}{270 \text{ lb.}}$$

is called a proportion.

c. The utility of a proportion comes from the fact that if only one of the numbers is not known it can easily be found. Suppose that two bombs are given, one weighing 450 lb., and the other weighing 150 lb., and that the length of the first bomb is 36 in. The length of bomb #2 is not known, but if the length of any bomb is "proportional" to its weight, then

$$\frac{\text{weight of bomb \#1}}{\text{weight of bomb \#2}}=\frac{\text{length of bomb \#1}}{\text{length of bomb \#2}}$$

is the proportion expressing this fact. Now some of these quantities are known:

$$\frac{450 \text{ lb.}}{150 \text{ lb.}}=\frac{36 \text{ in.}}{\text{length of bomb \#2}}$$

Therefore, if the proportion is true, then the length of bomb #2 must be 12 in.

d. In mathematics, not only are symbols such as $+$, $-$, $=$, and so on, used to simplify writing, but it is also convenient to introduce other symbols whenever they will shorten the work. Thus, to continue the preceding example, one might let

w_1=weight of bomb #1
w_2=weight of bomb #2
L_1=length of bomb #1
L_2=length of bomb #2

Then the proportion can be written even more simply as

$$\frac{w_1}{w_2}=\frac{L_1}{L_2} \text{ and } L_2=12 \text{ in.}$$

This practice of letting letters represent quantities is characteristic of all mathematics.

e. There is a general rule on proportions which can be stated very briefly in terms of symbols. Let a, b, c, and d be any quantities whatever. Then if there is a proportion between these quantities such that

$$\frac{a}{b}=\frac{c}{d},$$

then also

$$ad=bc.$$

That is to say, if the terms in a proportion be "cross-multiplied," then the products are equal:

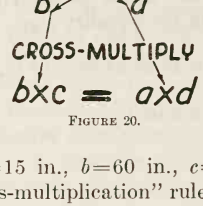


FIGURE 20.

(1) *Example:* Let $a=15$ in., $b=60$ in., $c=30$ yards, and $d=120$ yards. Check the "cross-multiplication" rule for these quantities.

Solution: Since $\frac{15 \text{ in.}}{60 \text{ in.}}=\frac{30 \text{ yd.}}{120 \text{ yd.}}$, then

(15 in.) (120 yd.) should equal (30 yd.) (60 in.).

(15 in.) (120 yd.)=1,800 in.–yd.=(30 yd.) (60 in.) *Check.*

f. A proportion need not be limited to the equality of only two ratios. Very often there will be three or more ratios that will be equal. For example, in the following triangles,

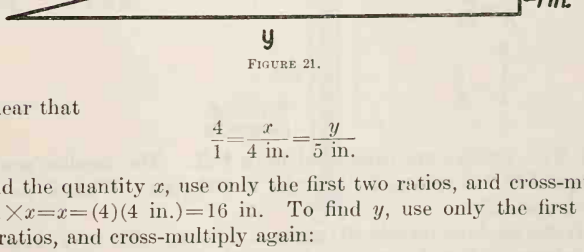


FIGURE 21.

it is clear that

$$\frac{4}{1}=\frac{x}{4 \text{ in.}}=\frac{y}{5 \text{ in.}}$$

To find the quantity x, use only the first two ratios, and cross-multiply: $1\times x=x=(4)(4 \text{ in.})=16 \text{ in.}$ To find y, use only the first and third ratios, and cross-multiply again:

$$y=(4)(5 \text{ in.})=20 \text{ in.}$$

g. Exercises.—It is customary to let x represent the unknown quantity. In the following exercises find the values of x which will make the proportions true.

(1) $\frac{5}{1}=\frac{x}{3}$

(2) $\frac{5}{6}=\frac{x}{3 \text{ in.}}$

Solution: Cross-multiplying, $6x=15$ in. If 6 x's are 15 inches, then obviously, one x must be one sixth as much or 2.5 inches. In all of these problems, if the factor multiplying x is not 1, divide by the factor on both sides of the equality sign. Thus

$$\frac{6x}{6}=\frac{15 \text{ in.}}{6} \text{ is the same as } 6x=15 \text{ in., but } 6x/6=x.$$

Therefore $x=(15 \text{ in.})/6=2.5$ in. *Answer.*

(3) $\frac{5}{3}=\frac{x}{5}$

(4) $\frac{3}{7}=\frac{5}{x}$ $x=11\frac{2}{3}$ *Answer.*

(5) $\frac{.25}{.75}=\frac{3}{x}$

(6) $\frac{(1/4)}{(.5)}=\frac{5}{x}$ $x=10$ *Answer.*

(7) $\frac{x}{y}=\frac{a}{b}$

(8) What is the value of A in the following proportions:

$$\frac{17}{45}=\frac{14}{\text{A}}$$

$$\frac{3}{\text{A}}=\frac{10}{25}$$ A=7.5 *Answer.*

$$\frac{\frac{3}{4}}{\frac{4}{5}}=\frac{6}{\text{A}}$$

(9) Two pulleys are connected by a belt. The smaller one runs at a speed of 750 rpm and the larger at 200 rpm. What is the ratio of their speeds?

(10) An airplane travels 400 miles in 2 hours. Set up a proportion and determine how far the airplane will travel in 14 hours.

2,800 mi. *Answer.*

(11) If a gear having 32 teeth makes 80 rpm, what will be the number of revolutions per minute of a smaller gear, which has 16 teeth?

(12) If a boat drifts down stream 40 miles in 12 hr., how far will it drift in 15 hr.? 50 mi. *Answer.*

(13) On June 12, 1939, a pilot flew a glider plane across Lake Michigan a total distance of 92 miles in 52 min. He cut loose from the tow plane at 13,000 ft. and descended only 5,000 ft. in crossing. At the same rate of descent, how much farther could he have glided? How many more minutes would he have been in the air?

(14) A roadbed rises $3\frac{1}{3}$ ft. in a horizontal distance of 300 ft. How many feet will the roadbed rise in 720 ft.? 8 ft. *Answer.*

(15) If 16 gal. of gas will drive a car 288 miles, at the same rate of using gas, how many gallons will it take to drive the same car from Chicago to Memphis, a distance of 564 miles?

14. Positive and negative numbers.—There are many quantities which by their contrary or opposite nature are best described as *negative* quantities in contrast to *positive* quantities. For example, temperatures above 0° Fahrenheit are considered as positive, whereas those below 0° are considered as negative. As a consequence it becomes necessary to consider negative and positive numbers and how to operate on them.

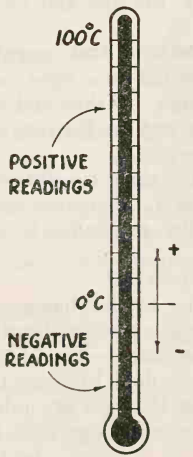


FIGURE 22.

a. A negative number is indicated by prefixing a minus sign (—) in front of the number. Thus, —5, —7.04, —90.003 are all negative numbers.

b. A positive number is indicated by prefixing it with a plus sign (+), if necessary. When there is no possible ambiguity the plus sign is usually omitted. Thus +5, 7.04, +63.0, 98.4 are all positive numbers.

c. It is convenient to imagine the numbers as representing distances along a straight line as follows:

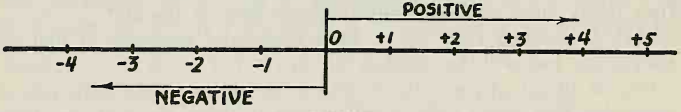


FIGURE 23.

Negative distances are measured to the left, and positive distances are measured to the right.

d. The signs + and — now have additional meanings. They not only indicate addition and subtraction, but positive and negative numbers as well. To distinguish the sign of operation from the sign of quality (positive or negative), the quality sign is enclosed in parentheses: 25 + (+5), 25 — (+5), 25 + (—5), or 25 — (—5). For the sake of brevity, the first and second are generally written simply as 25 + 5, and 25 — 5.

15. Addition of positive and negative numbers.—To add two numbers which have the same signs, add the numbers and prefix the common (or same) sign. If the numbers to be added have unlike signs, find the difference and use the sign of the larger number.

(1) *Example.* (+6)+(—3)=?

Solution: Since the signs are different, subtract the numbers to obtain a remainder of 3. Since the sign of the larger number is positive, the sign of the remainder is also positive: 6+(—3)=3 *Answer.*

(2) *Example:* (—3)+(+2)=?

Solution: Referring to the diagram in paragraph 14*c*, begin at —3 and count 2 units in a positive direction. The result is 1 space to the left of zero. Therefore (—3)+(+2)=—1.

This problem may also be done by using the rule stated in the preceding paragraph. Since the signs are unlike, subtract 2 from 3 and prefix the remainder by a minus sign since the larger number is negative. (—3)+(+2)= —1 *Answer.*

16. Subtraction of positive and negative numbers.—*a.* To subtract two numbers (positive or negative), change the sign of the number being subtracted and *add* the numbers as in addition (par. 15).

(1) *Example.* $(-3)-(-4)=?$

Solution. Changing the sign of the number being subtracted, the problem then becomes—

$(-3)+(+4)=+1$ *Answer.*

Referring to the diagram in paragraph 14*c*, this problem may also be done as follows: Begin at -4 and count in the opposite direction until -3 is reached. This requires 1 unit in the positive direction. Hence, $(-3)-(-4)=+1$ or in other words $+1$ must be added to -4 to give -3.

b. Exercises.

(1) $(-6)+(+6)=?$

(2) $(+2)+(+12)=+14$ *Answer.*

(3) $(-62)+(-18)=?$

(4) $(+17)-(+15)=+2$ *Answer.*

(5) $(+32)-(-64)=?$

(6) $(-18)-(-64)=+46$ *Answer.*

(7) $(-17)-(-15)=?$

(8) $(-17.3)+(35.4)=+18.1$ *Answer.*

(9) $(-17.36)-(35.4)=?$

(10) $(-201.03)-(-10.4)=-190.63$ *Answer.*

(11) $-18+.4=?$

(12) $20-17.4+9=+11.6$ *Answer.*

(13) $-37.3+19.4+17.8=?$

(14) $(-175.03)+19=-156.03$ *Answer.*

17. Multiplication and division of positive and negative numbers.—*a.* If the two numbers to be multiplied have the same signs, then the product is positive. If the two numbers to be multiplied have opposite signs, then the product is negative.

(1) *Example:* Multiply $(+3.04)$ by (17.8).

Solution: Since the signs are the same, the product is

$+54.112$ *Answer.*

(2) *Example:* $(+.00395)\times(-345.9)=?$

Solution: Since the signs are unlike, the product is negative, or -1.366305 *Answer.*

b. Exercises.—Find the product in each of the following exercises.

(1) $(-1.6)\ (.9)$

(2) $(-14.4)\ (-12)$ 172.8 *Answer.*

(3) $(12.5)\ (1.25)$

(4) $(-9)\ (-8)\ (-6)$ -432 *Answer.*

(5) $(-.17)(6)(-5)$
(6) $(2)(3.14)(9)$ 56.52 *Answer.*
(7) $(-.4)(-.4)(-.4)$
(8) $(-\frac{1}{2})(\frac{9}{20})(\frac{5}{6})$ $-\frac{1}{8}$ *Answer.*
(9) $(2\frac{2}{7})(1.4)(1.4)$
(10) $(-1\frac{2}{3})(\frac{9}{10})(-\frac{3}{15})$ $\frac{1}{5}$ *Answer.*

c. In division the quotient is positive if the divisor and dividend have the same sign; if the divisor and dividend have opposite signs, the quotient is negative.

(1) *Example:* Divide (-15.625) by (12.5)

Solution: Since the dividend and the divisor have opposite signs, the quotient is negative.

$$\frac{-15.625}{12.5} = -1.25 \qquad \textit{Answer.}$$

d. Exercises.—Find the quotient in each of the following exercises.

(1) $(-14.4) \div (0.9)$
(2) $(-4.32) \div (-4.8)$ 0.9 *Answer.*
(3) $(39{,}483.) \div (-12.3)$
(4) $(1{,}440.) \div (0.32)$ 4500. *Answer.*
(5) $(1.679) \div (23.)$
(6) $(-23.04) \div (4.8)$ -4.8 *Answer.*
(7) $\dfrac{1728}{-144}$
(8) $\dfrac{390.59}{-28.1}$ -13.9 *Answer.*
(9) $\dfrac{-72.9}{-0.81}$
(10) $\dfrac{0.6118}{87.4}$ 0.007 *Answer.*

18. Miscellaneous exercises.—The following exercises are based on the topics in section II:

(1) If 1 cu. ft. of water weighs 62.5 lb., what is the weight of 4.18 cu. ft. of water?

(2) How many cu. ft. are there in 180 lb. of water? (See ex. 1) 2.88 cu. ft. *Answer.*

(3) Change the following common fractions to decimal fractions:

$$\frac{3}{8};\ \frac{7}{16};\ \frac{51}{75};\ \frac{17}{32}.$$

(4) Change the following decimal fractions to mixed numbers:

$$1.25;\ 3.875;\ 14.375. \quad 1\frac{1}{4};\ 3\frac{7}{8};\ 14\frac{3}{8} \qquad \textit{Answer.}$$

(5) Bolts $\frac{3}{4}$ in. in diameter and 6 in. long weigh 117 lb. per hundred bolts. What is the weight of 1,200 bolts?

(6) A certain bomber can carry a bomb load of 4,500 lbs. How many 250 lb. bombs can be carried?

18 *Answer.*

(7) Which is larger $\frac{13}{15}$ or $\frac{23}{28}$?

(8) Divide $6\frac{3}{4}$ by $2\frac{5}{8}$.

$2\frac{4}{7}$ *Answer.*

(9) At an altitude of 5,000 ft. and at 10° C., the calibrated air speed is 190 mph. The true air speed is 206 mph. What is the percent of increase in the two readings?

(10) At an altitude of 11,000 ft. and at 20° C., the calibrated air speed is 210 mph. The true air speed is 242 mph. What is the percent of increase in the two readings? 15.2 percent *Answer.*

(11) The top air speed of an aircraft at 10,000 ft. is 325 mph. At 15,000 ft. it is 335 mph. What is the percent of increase in the air speed?

(12) 469 cadets are sent to primary schools in Georgia. This group represents 14 percent of the class. Find the number of cadets in the class. 3,350 cadets *Answer.*

(13) 28 cadets out of a squadron of 196 are on guard duty. What percent of the squadron is on guard duty?

(14) On a certain flight a bomber used 40.5 gal. of gasoline per hour. The time of the flight was 3 hr. 48 min. Find the amount of gasoline used. 153.9 gal. *Answer.*

(15) An aircraft flies a distance of 160 nautical miles. Find the distance in statute miles.

(16) The temperature reading on a Centigrade thermometer was 3° C. The reading increased 2° the first hour and decreased 7° the second hour. What was the final temperature reading?

−2° C. *Answer.*

(17) On a certain day, 10 temperature readings were taken on a Centigrade thermometer. They were 6°, −3°, −7°, −15°, −4°, 0°, 2°, 3°, 5°, 3°. Find the average temperature reading. *Hint:* Find the sum and divide by the number of readings.

(18) Find the product in each of the following:

$(-6).(-1\frac{1}{2}).(1\frac{1}{3})$ 12 *Answer.*

$(-2)^2.(8\frac{1}{3}).(-\frac{3}{4})$ −25 *Answer.*

(19) The following numbers represent the diameters of the bores on different guns: 37 mm, 3 in., 1 in., 155 mm, 6 in., 75 mm. Arrange them according to size beginning with the largest one.

(20) Express a speed of 118 kilometers per hour in terms of miles per hour. $73\frac{3}{4}$ mph *Answer.*

(21) Calculate the number of square centimeters in one square foot.

(22) A photographic film is designed for a picture 6 by 9 centimeters; express this in inches to the nearest quarter inch.

$2\frac{1}{4}$ by $3\frac{1}{2}$ in. *Answer.*

(23) 570 cadets are sent to primary schools in Florida. This group represents 30 percent of the class. Find the number of cadets in the class.

(24) 24 cadets out of a squadron of 180 passed the high altitude test. What percent of the squadron passed the test?

13.3 percent *Answer.*

(25) At a certain airdrome there are 88 aircraft, consisting of bombers and interceptor aircraft. The ratio of bombers to interceptors is 3 to 8. Find the number of each kind of aircraft.

(26) What is the diameter in inches of the bore of a 75-mm gun? (This means the bore is 75 mm in diameter.) 2.95 in. *Answer.*

(27) The following numbers represent the ranges of different aircraft: 250 nautical miles; 262 statute miles; 480 kilometers; 298 statute miles; 275 nautical miles. Arrange these distances in order of magnitude starting with the largest one.

(28) A detail of 33 cadets represents 15 percent of the squadron. How many cadets are there in the squadron? 220 *Answer.*

(29) Find the difference in temperature readings of $+47°$ C. and $-5°$ C.

(30) On a certain day the lowest temperature reading was $-14°$ F. and the highest temperature reading was $+19°$ F. Find the increase in readings. 33° *Answer.*

(31) Find the values of the following:

$(-3)^3$
$(-2)^2(-1)^3$
$3(-2)^3(-1)^2$
$2(\frac{1}{2})^2(4)^3$
$8(-1)^2(\frac{1}{2})^3$

(32) A panel is made up of 5 plies which are $\frac{1}{4}$ in., $\frac{3}{8}$ in., $\frac{1}{3}$ in., $\frac{1}{5}$ in., and $\frac{2}{15}$ in. thick, respectively. How thick is the panel?

$1\frac{7}{24}$ in. *Answer.*

(33) Divide 1.5625 by 0.125.

(34) Multiply $\left(2\frac{1}{4}\right)\left(\frac{8}{3\,6}\right)\left(2\frac{2}{3}\right)\left(1\frac{1}{8}\right)$. $1\frac{1}{2}$ *Answer.*

(35) Find the sum of $1\frac{1}{2}+2\frac{2}{3}-\frac{1}{4}+\frac{5}{1\,8}$.

(36) How many strips each $\frac{3}{3\,2}$ in. thick are in a laminated piece $1\frac{7}{8}$ in. thick? 20 *Answer.*

(37) In a squadron of 200 cadets there are 14 cadets sick. What percent of the squadron is sick?

(38) The chord of an airplane wing is 72 in. If the center of pressure is at a point 28 percent of the distance along the chord from the leading edge, how many inches is it from the leading edge?

20.16 in. *Answer.*

(39) The $\frac{L}{D}$ ratio for an airfoil section is $\frac{L_c}{D_c}$. Find the $\frac{L}{D}$ ratio when $L_c=0.0018$ and $D_c=0.00008$.

(40) Find the ratio of the areas of two circles having radii of 3 in. and 4 in. (The areas are to each other as the squares of their radii.)

$\frac{9}{1\,6}$ *Answer.*

Section III

EQUATIONS AND FORMULAS

	Paragraph
Purpose and scope	19
Equation	20
Axioms used in solving equations	21
Miscellaneous exercises	22

19. Purpose and scope.—The equation is the foundation of mathematics. In the solution of many problems, the first thing that is done is to write the given facts in the form of an equation. Formulas play an important part in all engineering and technical work. Laws and principles are written as formulas. In the handling of formulas the equation must be understood, for as soon as substitutions are made in a formula it becomes an equation. The following paragraphs contain exercises in setting up simple equations and handling formulas.

20. Equation.—An equation is a statement of equality between two equal numbers or symbols for numbers. For example, $17.5+.2=17.7$ means that the sum of 17.5 and .2 is 17.7, and $a=5b$ means that the number a is equal to five times the value of b. An equation of condition is an equation which is true only for certain values of a letter in it. For example, $3x-5=25$ is true only if $x=10$.

Exercises.—Write each of the following statements as an equation. In each case, state the meanings of your symbols or letters.

(1) The sum of five times a and two times b equals ten.

(2) Six times d diminished by nine equals fifteen.

$$6d-9=15 \qquad \textit{Answer.}$$

(3) The width of building is eight feet less than the length.

(4) The length of a rectangle exceeds the width by six feet.

$$W=L-6 \text{ or } W+6=L \ (W=\text{width}; L=\text{length}) \qquad \textit{Answer.}$$

(5) Three times a certain number exceeds the same number by twelve.

(6) A pilot flies an airplane a distance of 210 miles in 1 hour 12 minutes. Express average ground speed in miles per hour.

$$GS=\frac{d}{t}; GS=\frac{210 \text{ miles}}{1\frac{1}{5} \text{ hours}}; GS=175 \text{ miles per hour}$$

(GS=ground speed; d=distance; t=time) *Answer.*

(7) A pilot flies an airplane a distance of 350 miles in 1 hour 10 minutes. Express his average ground speed in miles per hour.

(8) The distance traveled equals the product of the average speed and the time. $d=vt$ *Answer.*

(9) The area of a rectangle equals the product of the length and the width.

(10) The product of the pressure exerted on a gas and the volume of the gas is a constant.

$$PV=K \qquad \textit{Answer.}$$

(P=pressure
V=volume
K=constant.)

21. Axioms used in solving equations.—The solution of equations depends on the following axiom, which includes four fundamental axioms (addition, subtraction, multiplication, and division) in one statement:

a. Axiom: If equal numbers are *added* to, *subtracted* from, *multiplied* by, or *divided* into equal numbers, the results are equal.

(1) *Example:* If $4x-5=2x+7$, what is x?

Solution: Subtracting $2x$ from each side of the equation, we have $2x-5=7$ (subtraction axiom)

Adding 5 to each side of the equation, we obtain $2x=12$ (addition axiom)

Therefore $x=6$ (division axiom) *Answer.*

b. The above equation may be solved by using so-called *transposition*, which is really just another name given to the addition and subtraction axioms. Any term may be *transposed* from one side of an equation to the other side, provided its sign is changed.

(1) *Example:* If $5x-3=2x+6$, what is x?
Solution: $5x-2x=6+3$ (transposing $2x$ and 3)
or $3x=9$
so that $x=3$ *Answer.*

c. These axioms will be used mainly to solve formulas for certain letters.

(1) *Example:* The lift of an airplane wing is given by the formula:

$$L=L_cAV^2$$

where
L=lift in pounds
L_c=lift coefficient
A=wing area in square feet
V=velocity in miles per hour

Solve this formula for A in terms of L, L_c, and V.

Solution: Using the division axiom and dividing each side of the equation by L_cV^2,

$$A=\frac{L}{L_cV^2}$$ *Answer.*

(2) *Example:* Given $C=\frac{5}{9}(F-32)$ where C=degrees Centigrade and F=degrees Fahrenheit. Solve for F.

Solution: $9C=5(F-32)$ (multiplication axiom)
or $9C=5F-160$
and $9C+160=5F$ (addition axiom)
so that $5F=9C+160$
and $F=\frac{9}{5}C+32$ (division axiom) *Answer.*

d. In the solution of word problems be sure that you write down the exact meaning of all the symbols that are used, as well as the units (feet, miles per hour, etc.).

(1) *Example:* A pursuit plane flies 90 mph faster than a bomber in still air. The bomber travels 42 miles while the pursuit plane travels 56 miles. Find the average speed of each aircraft.

Solution: Let r=average speed of the bomber in miles per hour; and $r+90$=average speed of the pursuit airplane in miles per hour. The two airplanes fly for the same period of time, hence

$$\frac{\text{distance pursuit airplane traveled}}{\text{speed of pursuit airplane}}=\frac{\text{distance bomber traveled}}{\text{speed of bomber}}$$

Substituting:

$$\frac{56}{r+90}=\frac{42}{r}$$

or

$$56r=42r+3{,}780$$

transposing, $56r-42r=3{,}780$
hence $14r=3{,}780$
and $r=270$ mph (speed of bomber).
consequently $r+90=360$ mph (speed of pursuit airplane)

Answer.

e. Exercises.

(1) The horsepower required for flying an airplane is found by the formula:

$HP=\frac{DV}{375}$, where HP=horsepower required
D=total drag of the airplane in pounds
V=velocity in miles per hour
and 375 is a constant

Find the horsepower required if $D=250$ lb. and $V=285$ mph.

(2) The general gas law is—

$\frac{PV}{T}=\frac{P_1V_1}{T_1}$ where P =initial pressure
V =initial volume
T=initial absolute temperature (273°+°C.)
P_1=new pressure
V_1=new volume
and T_1=new absolute temperature

Solve the above equation for V_1.

Solution: $\frac{PV}{T}=\frac{P_1V}{T_1}$

Multiply each side of the equation by TT_1:

$$PVT_1=P_1V_1T$$

or

$$P_1V_1T=PVT_1$$

Divide each side of the equation by P_1T:

$$V_1=\frac{PVT_1}{P\,T}$$

Answer.

(3) In exercise (2) if $P=15$ lb./sq. in., $T=7°$ C., $P_1=20$ lb./sq. in., $V_1=450$ cu. in., and $T_1=27°$ C., find the value of V.

(4) The formula $C=\frac{5}{9}(F-32)$ is used in the conversion of temperature readings from the Fahrenheit scale to the Cenitgrade scale. If the temperature reading is 86° on the Fahrenheit scale, what would the temperature reading be on the Centigrade scale?

Solution: $C=\frac{5}{9}(F-32)$
$C=\frac{5}{9}(86-32)$
$C=\frac{5}{9}(54)$
$C=30°$ *Answer.*

(5)(*a*) Using the formula in exercise (4) find the Fahrenheit reading when the Centigrade reading is 20°.

(*b*) When will the Fahrenheit and Centigrade readings be equal? (Negative values may be used.)

(6) The formula for determining the best propeller diameter for maximum efficiency is—

$$D=\frac{V}{1.03n}$$ where D=propeller diameter in feet
V=velocity of airplane in ft./sec.
n=revolution/sec
and 1.03 is a constant

Determine the propeller diameter when V=210 mph and n=30 revolutions/sec.

Solution: 210 mph $=\frac{(5280)(210)}{3600}$ ft./sec. $=308$ ft./sec.

$$D=\frac{308 \text{ ft./sec.}}{1.03\ (30 \text{ revolutions/sec.})}=9.97 \text{ ft.}$$ *Answer.*

(7) Using the formula in exercise (6), determine the propeller diameter when V=150 mph and n=20 revolutions/second.

(8) The horsepower necessary to drive an airplane is proportional to the cube of the velocity. If 120 hp is required to fly a plane at 130 mph, how many horsepowers would be required to fly it at 150 mph?

Solution: $\frac{HP_1}{HP_2}=\frac{V_1^3}{V_2^3}$
$\frac{120}{HP_2}=\frac{(130)^3}{(150)^3}$
$(130)^3.HP_2=120(150)^3$
$HP_2=\frac{120(150)^3}{(130)^3}$
$HP_2=184.3$ hp *Answer.*

(9) Referring to the information in exercise (8), how many times would the horsepower have to be increased to double the velocity?

(10) An airplane flying 160 mph covers a certain distance in 2 hr. 30 min. How long would it take it to cover the same distance when flying 200 mph?

2 hours *Answer.*

22. Miscellaneous exercises.— The following exercises are based on the material contained in section III:

(1) An airplane flying 120 mph covers a certain distance in 3 hr. 15 min. At what rate would it have to fly to cover the same distance in 2 hr. 30 min.?

(2) Find the value of V_1 in

$V_1=V_0(1+Kt)$ when $V_0=200$, $K=-.06$, and $t=15$.

20 *Answer.*

(3) Aircraft A flies west at 150 mph (ground speed); aircraft B takes off from the same point one hour later and flies west at 190 mph (ground speed). How long does B fly before he overtakes A?

(4) An aircraft interception listening post 50 miles away from an airdrome reports an enemy bomber directly over the post. The bomber is flying toward the airdrome at an estimated ground speed of 200 mph and at an altitude of 100 feet. If it takes an interceptor 3 minutes to take off and if he can fly at 300 mph ground speed, when and where will he intercept the enemy plane? (Assume the reported course and ground speeds remain constant.)

7.8 minutes after receiving report *Answer.*

24 miles from the airdrome *Answer.*

(5) An aircraft flies 1 hr. 30 min. at a certain average ground speed, and 2 hr. at 150 mph. The entire distance traveled was 630 miles. Find the average ground speed for the first part of the flight.

(6) A soldier walked 15 miles and then returned immediately in a "jeep" which averaged 45 mph. The entire trip required 4 hr. 5 min. Find the soldier's average rate of walking.

4 mph *Answer.*

(7) A quantity of air is enclosed in a vessel having a volume of 100 cubic centimeters. The pressure of the gas is 75 centimeters of mercury and its temperature is 27° C. Find the pressure exerted by the gas if its temperature is lowered to 0°C, and the volume is reduced to 30 cubic centimeters.

Hint: Let $V=100$ cc, $V_1=30$ cc, $T=300°$ (absolute), $T_1=273°$ (absolute), $P=75$ cm of mercury. Use the General Gas Law Formula:

$$\frac{PV}{T}=\frac{P_1V_1}{T_1}$$

(8) Two platoons of soldiers start on a hike walking at the rate of 4 mph. Two and one-half hours later a messenger is sent on a

bicycle traveling at the rate of 9 mph to call the platoons back. How soon will the messenger overtake the group?

2 hours. *Answer.*

(9) Members of the Baseball Boosters Association paid \$0.30 for admission to a baseball game and nonmembers paid \$0.50. Paid admissions numbered 278 and total receipts were \$97.80. How many nonmembers attended the baseball game?

(10) Eight cadets agree to buy a tent. Two of them find that they are unable to pay, so each one of the others has to pay \$4.00 more than he had expected to pay. What is the cost of the tent?

\$96.00 *Answer.*

(11) In curvilinear motion the centrifugal force in pounds is—

$C=\frac{W.V^2}{gr}$, where

W=the weight of the airplane in pounds
g=the acceleration of gravity (ft./sec./sec.)
V=the velocity of the airplane in ft./sec.
r=radius of curvature of the flight path in feet

Solve the above equation for r.

(12) In exercise (11) find the radius of curvature in feet if W=2,000 lb., g=32 ft./sec./sec., V=150 mph, and C=6,000 lb.

504 ft. *Answer.*

(13) An aircraft has a ground speed of 190 mph. What is its ground speed in knots? (1 knot=1.15 mph.)

(14) Rate of climb of an airplane in ft./min. is given by the formula

$$R=\frac{33000\ (\text{hp})}{W}$$

where (hp)=the excess horsepower available.
W=the weight of the aircraft in pounds,
and 33,000 is a constant. Find the rate of climb in ft./min. if (hp)=894 horsepower, and W=18,000 pounds.

1,639 ft./min. *Answer.*

(15) In exercise (14) find the rate of climb in ft./min. if (hp)=360 horsepower and W=6,000 pounds.

(16) The fuel consumption of a certain type of airplane engine is given by the formula

$$C=0.742(\text{hp})t$$

where C=the number of pounds of fuel consumed in t hours, and (hp)=horsepower. How many pounds of fuel will be used if an aircraft having two 750-horsepower engines makes a flight of 3 hours?

3,339 lb. *Answer.*

(17) Using the formula in exercise (16) how many gallons of fuel will be used by an aircraft having a 1,600-horsepower engine in a flight of 1 hr. 15 min.? (The weight of the fuel is 6 lb./gal.)

(18) The formula for computing the radius of action when returning to the same base is

$$R/A=\frac{(GS_1)(GS_2)T}{GS_1+GS_2}$$

where R/A=radius of action in miles
GS_1=ground speed out in mph
GS_2=ground speed back in mph
T=fuel hours minus 25 percent reserve.

Find the radius of action of an aircraft when $GS_1=140$ mph, $GS_2=100$ mph and $T=3$ hr.

175 miles *Answer.*

(19) An aircraft flies 330 miles in 2 hr. 12 min., using 61 gallons of fuel. What was its average speed in mph, and what was its fuel consumption in gallons/hour? miles/gallon?

(20) Using the formula given in example 21*c* (1), find the lift when $L_c=0.0048$, $V=180$ mph, and A=200 sq. ft.

31,104 lb. *Answer.*

Section IV

SCALES

	Paragraph
Scope	23
Models	24
Maps	25
Miscellaneous exercises	26

23. Scope.—The word scale is used in this section as in "*scale* model," the "*scale* of a map," the "*scale* of a drawing," and so on. The practical use of scales in connection with maps, drawings, and silhouettes is illustrated by examples and exercises.

24. Models.—*a.* A true scale model of an airplane, for example, is a model which has been constructed such that the ratio of the length of *any* part of the model to the actual length of the same part of the airplane is the same for all parts. Thus, if the wingspan on the model is 5 inches, and the wingspan of the actual airplane is 55 feet, then 1 inch anywhere on the model represents 11 feet or 132 inches. Then this is the *scale* of the model: 1 to 132, or $\frac{1}{132}$.

b. Example: The U. S. Government is encouraging youngsters to build scale models of various aircraft. The *scale* to be used is the same for all aircraft: 1 to 72. The wingspan of the German Heinkel bomber (He–111K Mk111) is 76 feet. What will be the wingspan of the model?

Solution: 76 feet $=76\times12$ inches

$$\frac{\text{wingspan (model)}}{76\times12 \text{ in.}}=\frac{1}{72}.$$

Therefore, wingspan of model $=\frac{76\times12}{72}=\frac{76}{6}=12\frac{2}{3}$ inches *Answer.*

c. Exercises.—The following models are all constructed to the scale of 1 to 72.

(1) The model wingspan of a B–18 is 15 inches. What is the actual wingspan?

(2) The over-all length of a Messerschmitt (Me–110) is 36 feet. How long will the model be?

6 in. *Answer.*

(3) The over-all length of a model of a B–23 is 8⅝ inches. What is the over-all length of the B–23 airplane?

25. Maps.—*a.* A map is a scale diagram to show the disposition of geographic features on the earth such as cities, roads, rivers, and so on. On most maps, the scale used is conveniently stated by a diagram as in figure 24.

FIGURE 24.

It may also be expressed as a ratio: 1 to 500,000, for example.

b. Of primary interest to the pilot are aeronautical charts, or maps. The sectional charts of the United States are made at a scale of 1 to 500,000. The regional charts of the United States are made with a scale of 1 to 1,000,000.

(1) *Example:* What distance (miles) does 1 inch represent on a sectional chart?

Solution: Since the scale is 1 to 500,000, then 1 inch represents 500,000 inches on the earth. 1 mile $=5280$ feet $=5280\times12$ inches. Therefore

$$500{,}000 \text{ in.}=\frac{500{,}000}{5{,}280\times12} \text{ miles}=7.9 \text{ miles}.$$

1 inch represents 7.9 miles *Answer.*

26. Miscellaneous exercises.

(1) 1 inch on a regional chart=how many miles?

(2) The aeronautical planning chart of the United States (3060*a*) is drawn at a scale of 1 to 5,000,000. On this chart, a distance of 2¼ inches is the same as how many miles? 181.1 mi. *Answer.*

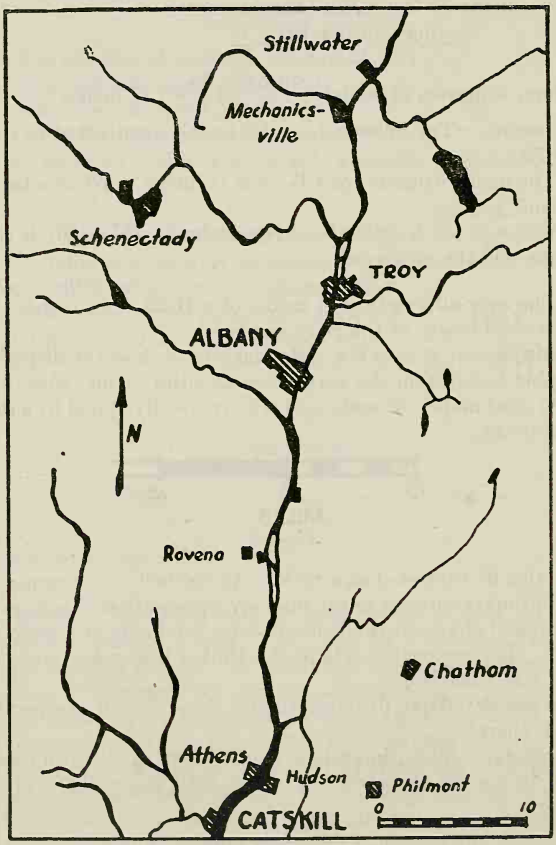


FIGURE 25.

(3) By direct measurement determine the scale for the map in figure 25. From Catskill to Albany is 30 miles.

(4) How far is Schenectady from Albany? See figure 25, and use the scale determined in exercise (3). 16 mi. *Answer.*

(5) By direct measurement, determine the scale used for the map in figure 26. See exercise (3).

(6) Is Chatham located properly in figure 26? Why? Its location is correct in figure 25. No *Answer.*

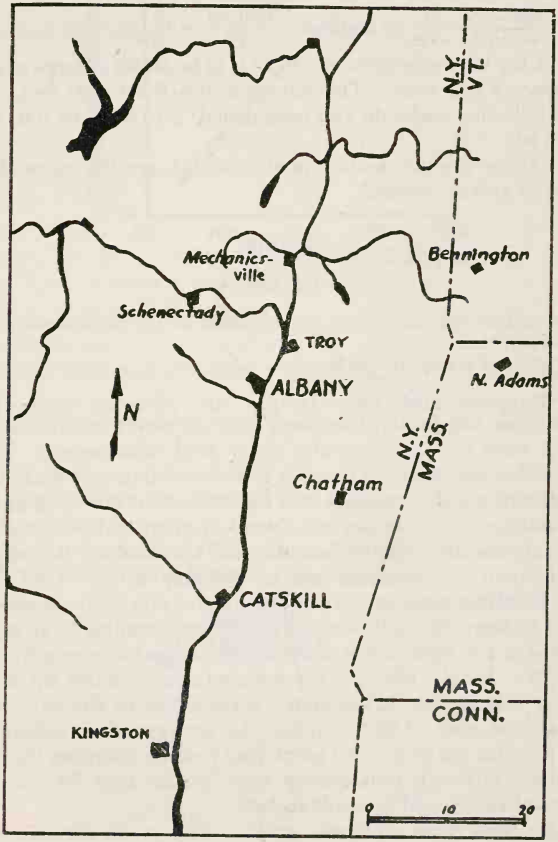


FIGURE 26.

(7) The model of the German bomber Heinkel (He–177) has a wingspan of 17¼ inches. What must be its actual wingspan?

(8) The airplane models mentioned in paragraph 24*b* will be used for training gunners in range determination. How far from the model

should a gunner be so that it will appear to him as though the actual airplane were 600 yards away?

Solution: His distance from the model should be in the same ratio to 600 yards as the model scale, or as 1 is to 72:

$$\frac{\text{distance}}{600 \text{ yd.}}=\frac{1}{72}, \text{ or distance}=\frac{600 \text{ yd.}}{72}=8\frac{1}{3} \text{ yd.}=25 \text{ feet } \textit{Answer.}$$

(9) A top view silhouette of a B–23 is to be drawn as large as possible in a space 2 feet wide. The wingspan of a B–23 is 91 feet. Which of the following scales do you recommend: 1 to 60, 1 to 100, 1 to 20, or 1 to 50?

(10) Using the information in exercise (3), are the scales shown in figures 25 and 26 correct?

Section V

GRAPHS

Paragraph
Purpose and use 27
Reading graphs 28
Construction of graphs from data 29

27. Purpose and use.—Graphs are pictorial representations showing the relationships between two (or more) quantities. They have a wide use in practically every field of endeavor. In aeronautics they are used for showing experimental or test data, to show the calibration of instruments, and for saving time and work in making computations. Graphs are also found in many publications such as technical manuals, scientific journals, and text books. It is important that the pilot trainees know how to read graphs.

28. Reading graphs.—To illustrate the procedure, suppose it is desired to know the calibrated air speed corresponding to an indicated air speed of 150 mph on the meter whose calibration curve is shown in figure 27. From "150" on the horizontal axis, move up until the curve is intersected. Then move horizontally to the left. On the vertical axis read "158." This is the corresponding calibrated air speed in miles per hour. All graphs are read by following this general procedure, although occasionally some graphs may be complicated by several curves and multiple scales.

a. Air speed meter calibration graph.—This graph (fig. 27) is based upon an air speed meter calibration of an airplane.

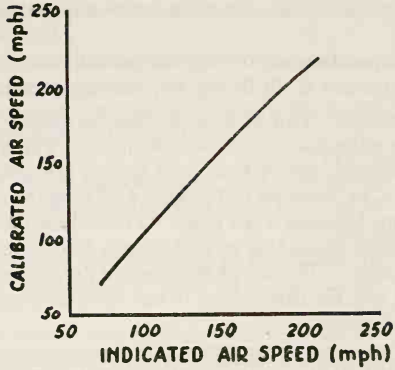


FIGURE 27.—An air speed meter calibration curve.

(1) If the indicated air speed is 110 mph, find the calibrated air speed.

(2) If the indicated air speed is 165 mph, find the calibrated air speed. 173 mph *Answer.*

(3) If the calibrated air speed is 198 mph, find the indicated air speed.

b. Pressure-temperature graph.—This graph (fig. 28) shows the relationship between the pressure and the temperature of a gas at constant volume.

(1) At a temperature of 0° C., the pressure is _____.

(2) If the temperature increases, the pressure _____.

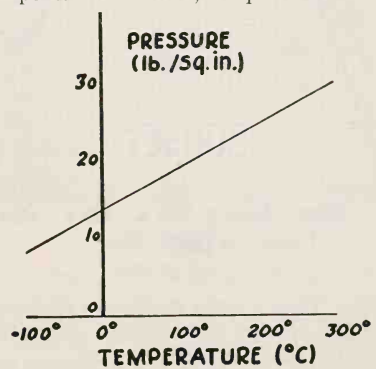


FIGURE 28.—Pressure and temperature of a gas at constant volume.

(3) If the pressure is decreased, the temperature of the gas _____.

(4) At a temperature of 100° C., the pressure is _____.

(5) If the pressure is 30 lb./sq. in., the temperature is _____.

c. 24-hour system.—The 24-hour system for stating time eliminates the use of the abbreviations a. m. and p. m. The values for a. m. time are unchanged except that four figures are always used. For example, 9:15 a. m. becomes 0915 hour; 4:45 a. m. becomes 0445 hour and 11:48 a. m. becomes 1148 hour. The values for p. m. time are increased by 1200, hence 1:30 p. m. becomes 1330 hour and 5:55 p. m. becomes 1755 hour. The use of this system decreases the chances for making errors and for this reason it has been adopted for use in the U. S. Army Air Forces.

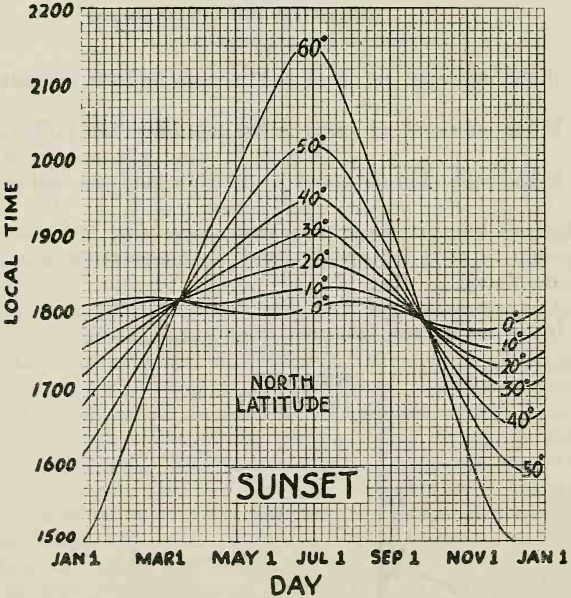


FIGURE 29.—Sunset graph.

d. Sunset graphs.—These graphs enable the pilot to determine the time of sunset for any position on the earth. The 24-hour system of keeping time is used in sunset graphs.

(1) *Instructions for use.*—(*a*) Enter the top or bottom scale with proper date.

(*b*) Move vertically down or up to the curve for observer's latitude (observer's position).

(*c*) Move horizontally to the right or left and read local civil time of sunset on vertical scales at the side.

(2) Find the sunset time for Nov. 1 at latitude 30° N.
(Follow instructions for use.)

1727 hour *Answer.*

(3) Find the sunset time for May 15 at latitude 50° N.

(4) Find the sunset time for May 20 at latitude 30° N.

1852 hour *Answer.*

(5) Find the sunset time for June 10 at latitude 10° N.

(6) Find the sunset time for Feb. 10 at latitude 40° N.

1736 hour *Answer.*

(7) Find the sunset time for Oct. 20 at latitude 30° N.

e. Fuel consumption graph.—This graph (fig. 30) shows the relationship between the air speed and the fuel consumption.

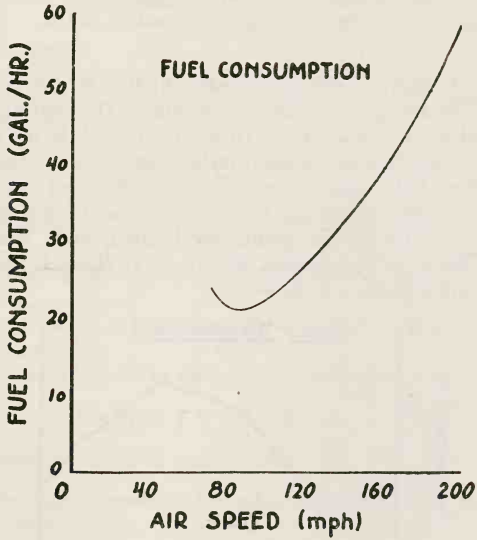


FIGURE 30.—Typical fuel consumption curve.

(1) At an air speed of 180 mph, the fuel consumption is ___ gal./hr.
(2) At an air speed of 168 mph, the fuel consumption is ___ gal./hr.
(3) If the fuel consumption is 53 gal./hr., the air speed is____ mph.
(4) If the fuel consumption is 39 gal./hr., the air speed is ____ mph.

29. Construction of graphs from data.—*a.* Neatness is important in the construction of graphs. The work should be carefully planned first. The choice of a suitable scale should be made by observing the range of the data. The axes should be labeled according to what they represent and the graph should have a title.

(1) *Example:* Plotagraph of the following temperature data:

Time	Temperature (degrees F.)	Time	Temperature (degrees F.)
0100	60	1300	72
0200	60	1400	73
0300	59	1500	73
0400	58	1600	74
0500	57	1700	73
0600	57	1800	71
0700	60	1900	70
0800	63	2000	68
0900	65	2100	67
1000	68	2200	65
1100	70	2300	62
1200	71	2400	61

(*a*) *Hint:* Choose a convenient scale so that all the data can be represented on the graph without crowding. The horizontal axis is usually used as the time axis. To locate the points for the graph, follow the vertical line indicating 0100 hour to its intersection with the horizontal line indicating a temperature of 60° and place a point there. Repeat this procedure for each hour and its corresponding temperature. After all the points are located, draw straight lines connecting them, or if possible a smooth curve through the points.

(*b*) A solution is given in figure 31.

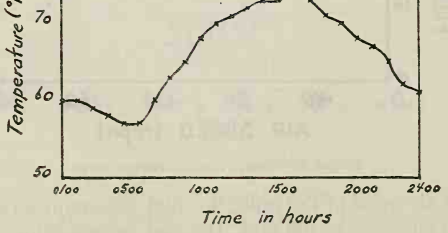


FIGURE 31.

b. Exercises.—(1) The height H, in yards, of a light above sea level and the distance D, in miles, at which it can be seen from the surface of the sea are given in the table. Plot the graph.

D	0	3	6	12	18	24	36	45
H	0	2	8	32	72	128	288	450

The height a light must be above sea level to be seen 30 miles out is _____ yards.

(2) Plot calibrated air speed in knots against indicated air speed in mph.

Calibrated air speed (knots)	101	110	119	128	137	146
Indicated air speed (mph)	110	120	130	140	150	160

If the indicated air speed is 145 mph, the calibrated air speed in knots is _____. If the calibrated air speed in knots is 130, the indicated air speed in mph is _____.

(3) The length of a pendulum and the time it requires to make a complete vibration are given in the following table. Represent this data graphically.

Length in centimeters	50	60	70	80	90	100	110
Time in seconds	1.42	1.55	1.68	1.80	1.90	2.01	2.11

(*a*) The time of one vibration of a pendulum 65 cm long is _____ sec.

(*b*) The length of a pendulum which makes a complete vibration once in 1.5 seconds is _____ cm.

(4) Plot true air speed in mph against calibrated air speed in mph.

True air speed in mph	110	125	141	151	164
Calibrated air speed in mph	100	114	128	137	149

(*a*) If the true air speed is 130 mph, then the calibrated air speed is _____ mph.

(*b*) If the true air speed is 160 mph, then the calibrated air speed is _____ mph.

(*c*) If the calibrated air speed is 110 mph, then the true air speed is _____mph, _____ knots.

(5) Plot centigrade temperatures against Fahrenheit temperatures. A table showing the relationship between Fahrenheit and Centigrade temperature readings is given.

Temperature in degrees Fahrenheit	32	68	104	140	176	212
Temperature in degrees centigrade	0	20	40	60	80	100

(*a*) A temperature of 96° F.=_____° C.
(*b*) A temperature of 15° C.=_____° F.

Section VI

ANGULAR MEASUREMENT

Paragraph
Purpose 30
Angle and units of angular measurement 31
Course, heading, drift 32
Exercises 33

30. Purpose.—The purpose of this section is to familiarize the pilot trainee with angular measurements and how they are used in determining directions in the United States Army Forces.

31. Angle and units of angular measurement.—*a.* Consider a circle. The circumference, or any part of that circumference which is called an arc, is divided into units called degrees. There are 360 degrees in a complete circumference (or one revolution). For more accurate measurements a degree is divided into 60 equal parts called minutes and a minute is divided into 60 equal parts called seconds. The following table shows the symbols used for these units of angular measurement.

(1) Table of angular measurement.

360° (degrees)=1 circumference

$1°=\frac{1}{360}$ part of a circumference

60′ (minutes)=1°

60″ (seconds)=1′

Only the degree will be used in this manual.

b. An angle is the figure formed by drawing two straight lines outward from a common point. The common point is called the vertex of the angle and the straight lines are called the sides of the angle. In figure 32, O is the vertex and NO and PO are the sides of the angle NOP. Another definition of an angle is the amount of rotation or turning necessary to rotate NO to the new position PO. The air

navigation system of measuring and naming directions consists of designating directions by measuring them in degrees clockwise from the north through 360°.

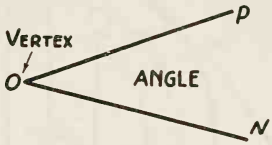


FIGURE 32.

c. The instrument for measuring angles is the protractor. To measure an angle with a protractor, place the protractor on the angle to be measured (see fig. 33) so that either half of the side AB will fall upon one side of the angle and the point O on the vertex. The reading on the scale where the other side of the angle crosses it is the measure of the angle in degrees.

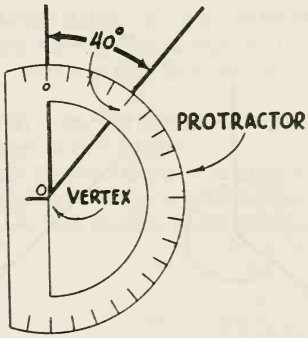


FIGURE 33.

d. Exercise.—Measure each of the angles in figures 34, 35, 36, and 37.

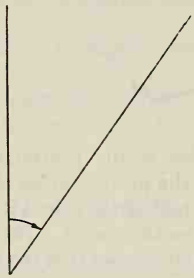


FIGURE 34.

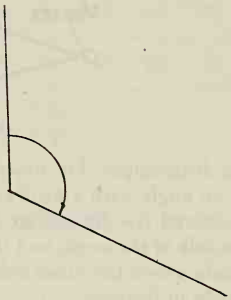


FIGURE 35.

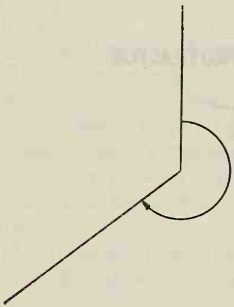


FIGURE 36.

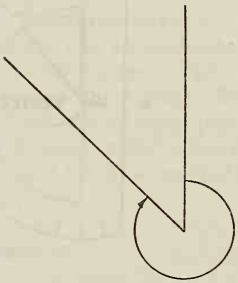


FIGURE 37.

e. To lay off an angle with a protractor.—Draw one side of the angle and locate the vertex. Place side AB of the protractor on the side drawn and point O on the vertex. Locate the reading of the value of the angle required on the scale of the protractor and connect this with the vertex. Measure the angle by starting at the north line.

(1) *Example.* Lay off an angle of 115°.

Solution: First draw the north line, then follow the instructions in *e* (see fig. 38).

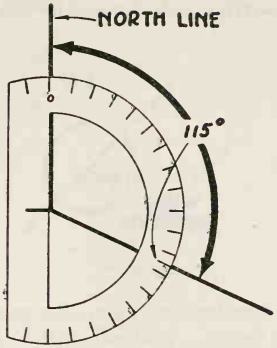


FIGURE 38.

(2) *Exercise.*—Use a protractor and lay off angles of 30°, 135°, 180°, 240°, 315°.

32. Course, heading, drift.—*a. True course* (*C*)—Direction over the surface of the earth expressed as an angle with respect to true north in which an aircraft intends to fly. It is the direction as laid out on a map or chart.

b. True heading (*H*)—Angular direction of the longitudinal axis of the aircraft with respect to true north.

c. Drift (*D*)—Angle between the true heading and the true course. It is *right* drift if the true course is *greater* than the true heading. If the true course is *less* than the true heading, it is *left* drift.

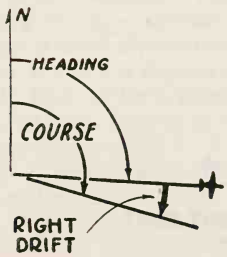


FIGURE 39*a*.

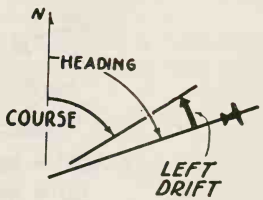


FIGURE 39*b*.

d. Wind direction.—Wind is designated by the direction *from* which it blows (see fig. 40).

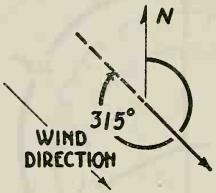


FIGURE 40.

e. In determining true heading when true course is given, subtract right drift from true course. Add left drift to true course to obtain true heading.

33. Exercises.—*a.* Determine the direction (north, south, east, west, etc.) in each of the following cases.

(1) True course 180°
(2) True course 270° True course west *Answer.*
(3) True course 225°
(4) Wind from 315° Wind from northwest *Answer.*
(5) Wind from 135°
(6) Wind from 0° Wind from north *Answer.*
(7) True heading 45°
(8) True heading 90° True heading east *Answer.*

b. Determine the true heading in each of the following cases:

(1) True course=90°
Right drift=6°
Solution:

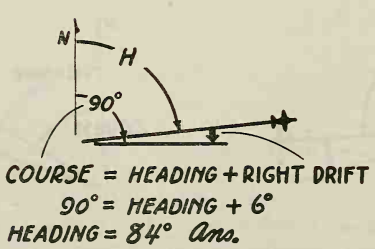


FIGURE 41.

(2) True course=135°
Left drift=9° True heading=144° *Answer.*

(3) True course=270°
Left drift=11°
(4) True course=315°
Right drift=10° True heading=305° *Answer.*
(5) True course=0°
Left drift=15°

SECTION VII

VECTORS

Paragraph
Purpose 34
Vector 35
Triangle of velocity 36
Miscellaneous exercises 37

34. Purpose.—The purpose of this section is to give the pilot trainee an idea of what a vector is and of how vectors will be used in triangle of velocity problems.

35. Vector.—*a.* A velocity is a rate of change of position in a given direction. A velocity is made up of both speed and direction. The statement that an aircraft has a velocity of 250 mph is incomplete without an indication of the direction of flight.

b. A vector is a straight line with an arrow showing direction. Since a velocity implies a speed (air speed, ground speed or speed of wind) in one direction, it may be represented graphically by a straight line whose length is drawn to some scale, for example, 1 inch=20 mph, and whose direction is measured in degrees clockwise from north. Thus the graphical representation of a velocity is a vector.

c. Vector addition.—The addition of two or more vectors yields a vector which is equivalent to the other vectors. This equivalent vector is called the resultant. To find the sum of two or more vectors, say a, b, and c, a polygon is constructed as follows:

(1) Each vector is placed so that its initial point is in contact with the end point (arrow) of its predecessor. The vectors may be added in any order.

(2) The resultant is the vector joining the first initial point to the last end point.

FIGURE 42.

36. Triangle of velocity.—*a.* An aircraft in flight follows a direction relative to the ground which in general is not the same as the direction in which the aircraft is headed. The actual path of an aircraft in flight over the earth's surface is known as its track. If there is no wind then the aircraft flies in the direction in which it is headed and its track is in the same direction. Due to the general prevalence of the wind, an aircraft is rarely headed in the direction of its track.

b. In effect an aircraft is subject to two velocities:

(1) Wind velocity, and

(2) Its velocity through the air mass surrounding it. This second velocity is determined by two factors: the true air speed, and the direction in which the aircraft is headed. The resultant of these two velocity vectors is a vector in the direction of the track (course) whose length is the ground speed.

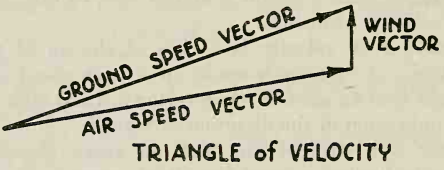


FIGURE 43.

c. *Example:* Given: Wind 30 mph from 200°
Air speed 150 mph
Heading 90°

Required: Ground speed and course.

Solution:

(1) Draw the north line.

(2) Draw the air speed vector at an angle of 90° from north, making it 5 in. long (1 in.=30 mph; hence 5 in.=150 mph).

(3) Draw the wind vector away from 200°, making it 1 in. long (1 in.=30 mph). The wind vector is drawn from the end point of the air speed vector. (Air speed vector+wind vector=ground speed vector.)

(4) The angle between the ground speed vector and the north line determines the course. The length of the ground speed vector according to the scale used determines the ground speed.

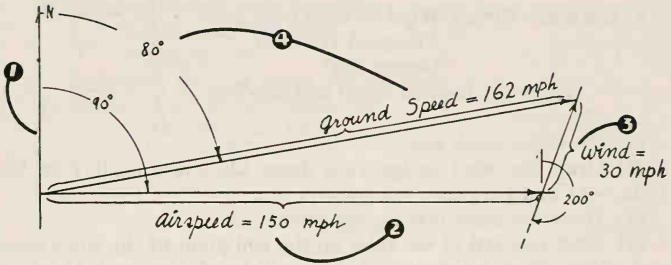


FIGURE 44.

(5) *Remark.*—The drift angle is the angle between the air speed vector and the ground speed vector. Since this is a case of left drift, the drift angle of 10° is subtracted from the heading 90° to obtain the course 80°.

d. Exercises.

(1) Given: Wind 30 mph from 315°
Air speed 210 mph
Heading 160°
Required: Ground speed and course.

(2) Given: Wind 30 mph from 180°
Air speed 150 mph
Heading 90°
Required: Ground speed and course.
Ground speed = 153 mph
Course = 79° *Answer.*

(3) Given: Wind 40 knots from 210°
Air speed 160 knots
Heading 270°
Required: Ground speed and course.

(4) Given: Wind 40 mph from 90°
Air speed 180 mph
Heading 215°
Required: Ground speed and course.
Ground speed = 206 mph
Course = 224° *Answer.*

(5) Given: Wind 45 knots from 300°
Air speed 180 knots
Heading 45°
Required: Ground speed and course.

e. Example: Given: Wind 35 mph from 225°
Air speed 175 mph
Course 300°
Required: Ground speed and heading.
Solution:

(1) Draw the north line.

(2) Draw the wind vector away from 225°, making it 1 in. long (1 in.=35 mph).

(3) Draw the track making the course 300°.

(4) Place one end of the ruler on the end point of the wind vector and notice where a vector 5 in. long will touch the track (1 in.=35 mph; hence 5 in.=175 mph).

(5) Draw the air speed vector as determined in (4).

(6) The length of the track vector according to the scale used determines the ground speed.

(7) The angle between the air speed vector and the north line measured clockwise from north determines the heading.

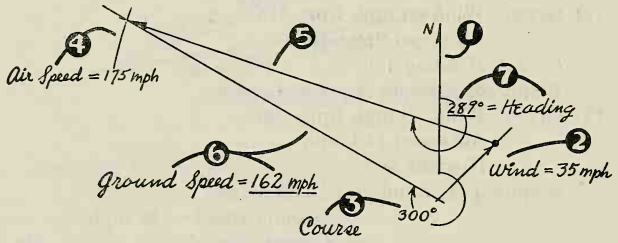


FIGURE 45.

(8) *Remark.*—The drift angle is the angle between the air speed vector and the ground speed vector. Since this is a case of right drift, the drift angle of 11° is subtracted from the course 300° to obtain the heading 289°.

f. Exercises.

(1) Given: Wind 50 mph from 300°
Air speed 225 mph
Course 270°
Required: Ground speed and heading.

(2) Given: Wind 25 knots from 150°
Air speed 150 knots
Course 345°
Required: Ground speed and heading.
Ground speed=174 knots
Heading=348° *Answer.*

(3) Given: Wind 35 mph from 0°
Air speed 210 mph
Course 120°
Required: Ground speed and heading.

(4) Given: Wind 55 mph from 120°
Air speed 220 mph
Course 60°
Required: Ground speed and heading.
Ground speed=187 mph
Heading=73° *Answer.*

(5) Given: Wind 10 knots from 135°
Air speed 165 knots
Course 315°
Required: Ground speed and heading.

37. Miscellaneous exercises.—The following exercises are to be solved by graphical methods and include both types of triangle of velocity problems discussed in paragraph 36.

(1) Given: Wind 25 mph from 240°
Air speed 175 mph
Heading 120°
Required: Ground speed and course.

(2) Given: Wind 15 knots from 135°
Air speed 195 knots
Course 135°
Required: Ground speed and heading.
Ground speed=180 knots
Heading=135° *Answer.*

(3) Given: Wind 35 mph from 30°
Air speed 245 mph
Course 180°
Required: Ground speed and heading.

(4) Given: Wind 30 mph from 270°
Air speed 242 mph
Heading 324°
Required: Ground speed and course.
Ground speed=223 mph
Course=330° *Answer.*

(5) Given: Wind 40 mph from 60°
Air speed 230 mph
Course 0°
Required: Ground speed and heading.

APPENDIX

MISCELLANEOUS UNITS AND CONVERSION FACTORS

1. Formulas.

a. Area of circle, diameter $=d$, is $\frac{d^2}{4}$.

b. Area of rectangle, width$=w$ and length $=l$, is wl.

c. Area of triangle, base$=b$ and altitude $=h$, is $bh/2$.

d. Volume of sphere, radius $=r$, is $\frac{4}{3}\pi r^3$.

e. Volume of prism, area of base$=A$, height$=h$, is Ah.

2. Conversion factors.

66 nautical miles	=76 statute miles=122 kilometers (approximate)
1 centimeter (cm)	=0.393700 inches (in.)
1 foot (ft.)	=12 inches (in.)
	=30.4801 centimeters (cm)
1 inch (in.)	=2.54001 centimeters (cm)
1 kilometer (km)	=1,000 meters (m)
	=0.62137 statute mile (mi., stat.)
	=0.53959 nautical mile (naut.)
	=3280.83 feet (ft.)
1 meter (m)	=100 centimeters (cm)
	=1,000 millimeters (mm)
	=3.28083 feet (ft.)
	=39.3700 inches (in.)
1 millimeter (mm)	=0.039370 inches (in.)
1 nautical mile (naut.)	=6,080.20 feet (ft.)
	=1.151553 statute miles (mi., stat.)
	=1.853249 kilometers (km)
1 statute mile	=5,280 feet (ft.)
	=1.60935 kilometers (km)
	=.868393 nautical miles
1 U. S. gallon (gal.)	=231 cubic inches (cu. in.)
	=.13368 cubic feet (cu. ft.)
	=.83310 British gallons
1 cubic foot (cu. ft.)	=1,728 cubic inches (cu. in.)
	=7.4805 U. S. gallons (gal.)

1 kilometer per hour (km/hr)	=0.62137 statute miles per hour
	=0.53959 knots
1 knot	=1 nautical mile per hour
	=1.853249 km/hr
	=1.151553 mph or mi/hr
1 statute mile per hour (mph or mi/hr)	=1.4666 feet per second
	=1.60935 km/hr
	=0.868393 knots

3. Temperature scales.

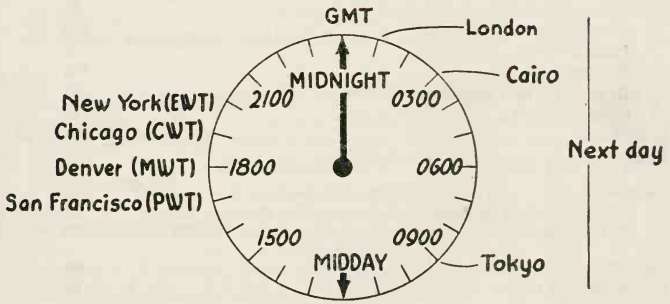
Freezing: 0° C. =32° F. =273° K. (Absolute)
Boiling: 100° C. =212° F. =373° K. (Absolute)

$$C° = \frac{5}{9}(F° - 32).$$

$$F° = \frac{9C°}{5} + 32.$$

$$K° = C° + 273.$$

4. Time equivalents.



5. Densities.

Gasoline (aviation) weighs 45 lb./cu. ft. or 6 lb./gal.
Oil (aviation) weighs 56 lb./cu. ft. or 7.5 lb./gal.
Water weighs 62.4 lb./cu. ft. or 8.34 lb./gal.
Air (dry) weighs .0765 lb./cu. ft. at 15° C. (59° F.) and standard atmospheric pressure.

INDEX

	Paragraph	Page
Addition	4*a*	2
Exercises	4*d*	3
Of fractions	10*a*	19
Of positive and negative numbers	15	28
Air navigation system of measuring directions	31*b*	50
Exercises	33*a*, 33*b*	54
Air speed meter calibration graph	28*a*	44
Air speed vector	36*c*(2), 36*c*(5)	56, 58
Angle	31*b*	50
Construction of	31*e*	52
Angular measurement	31	50
Exercises	31*d*	52
Table of	31*a*(1)	50
Calibrated air speed	29*a*(2), 29*a*(4)	49
Centigrade scale	21*c*, 21*e*(4), 29*a*(5)	35, 37, 48
Centrifugal force	22(11)	38
Common denominator	10*c*	20
Course	36*c*(4)	56
Decimal fractions	8*a*	14
Decimal point	4*a*, 6*b*, 7*f*	2, 6, 11
Dividend	7*a*, 7*f*	9, 11
Divisor	7*a*	9
Division	7*a*	9
Exercises	17*d*	30
Long division	7*c*	9
Of fractions	12*a*	22
Of positive and negative numbers	17*c*	30
Short division	7*b*	9
Drift angle	36*c*(5), 36*e*(8)	57, 58
Drift	32*c*	53
Equations	20	33
Axioms used in solving	21	34
Formation of	20	33
Factors	6*a*	5
Rounding off	6*d*	7
Fahrenheit scale	21*c*, 21*e*(4), 29*a*(5)	35, 37, 48
Fractions:		
Addition of	10*a*	19
Conversion of	8*a*, 9*a*	14, 18
Division of	12*a*	22
Exercises	8*d*, 8*f*, 9*d*, 10*d*, 11*c*, 12*c*	15, 17, 19, 20, 22, 23
Multiplication of	11*a*	21
Reduction to lowest terms	8*c*	15
Subtraction of	10*b*	20

INDEX

	Paragraph	Page
Fuel consumption	22(16)	39
Graph of	28*e*	47
General gas law	21*e*(2)	36
Graphs	27	44
Construction of	29	48
Reading of	28	44
Ground speed vector	36*c*(4), 36*e*(6)	56, 58
Horsepower	21*e*(1), 21*e*(8)	36, 37
Indicated air speed	29*b*(2)	49
Lift	21*c*(1)	35
Maps	25*a*	41
Materials needed	2	2
Miscellaneous exercises—		
On equations	22	38
On fundamental operations	18	30
On scales	26	42
On vectors	37*a*	59
Mixed numbers	7*h*	12
Models	24	40
Multiplication	6*a*	5
Exercises	6*g*, 6*h*, 17*b*	8, 29
Of fractions	11*a*	21
Of positive and negative numbers	17*a*	29
Percent	8*e*	16
Positive and negative numbers	14	27
Addition of	15*a*	28
Division of	17*c*	30
Exercises	16*b*, 17*b*, 17*d*	29, 30
Multiplication of	17*a*	29
Subtraction of	16*a*	29
Pressure-temperature graph	28*b*	45
Proportion	13*b*	24
Exercises	13*g*	26
Protractor, use of	31*c*	51
Quotient	7*a*	9
Radius of action	22(18)	40
Rate of climb	22(14)	39
Ratio	13*a*	23
Regional charts	25*b*	41
Resultant	36*b*(2)	56
Scales	23	40
Miscellaneous exercises	26	42

INDEX

	Paragraph	Page
Sectional charts	25*b*	41
Subtraction	5*a*	4
Exercises	5*e*	5
Of fractions	10*b*	20
Of positive and negative numbers	16*a*	29
Sunset graphs	28*d*	46
Symbols	4*c*, 6*e*, 7*e*, 13*d*, 14*d*	3, 7, 11, 24, 28
Temperature conversion formula	21*c*, 21*e*(4)	35, 37
Transposition	21*b*	35
Triangle of velocity	36*a*	56
Exercises	36*d*, 36*f*	57, 58
Miscellaneous exercises	37	59
True air speed	29*b*(4)	49
True course	32*a*	53
True heading	32*b*	53
24-hour system	28*c*	46
Units	4*b*, 6*e*, 7*j*	3, 7, 13
Vectors	35*a*	55
Addition of	35*c*	55
Wind direction	32*d*	54
Wind vector	36*c*(3), 36*e*(2)	56, 58

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(For explanation of symbols see FM 21–6.)

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